The Arithmetic Teacher

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Time Spent on Arithmetic in Foreign Countries and in the United States
G. R. MILLER

Number, Numeral, and Operation
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The Slow Can Learn
MARY A. POTTER

The Relationship of Socio-Economic Factors and Achievement in Arithmetic
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"Plus" Work for All Pupils
DAVID M. CLARKSON

THE ARITHMETIC TEACHER

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THE ARITHMETIC TEACHER

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May 1960

Time Spent on Arithmetic in Foreign Countries and in the United States

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Although many instructors are aware of the time spent on arithmetic in the U.S.A., few would probably know how much time is spent on arithmetic in other nations of the world. Those of you with inquiring minds would probably ask several questions. For instance: Do other nations devote more time, the same time, or less time for the study of arithmetic than we provide in this country? If there are any differences, what are they? Does increased time spent in arithmetic produce a difference in ability of students in different nations? The first two questions can be answered by means of available statistical data. The examination of the time spent in arithmetic and its possible effects on student ability is difficult because of the complex differences in the school systems considered.

Background Information

In the previous article, "How Much Time for Arithmetic?" (The Arithmetic Teacher, November, 1958), the time allotments for arithmetic in each grade level for the schools of the United States were evaluated. In order to assess the results of the present study, the conclusions of this previous article will be stated. These were:

1. The amount of time spent on arithmetic ranged from a median score of 23 minutes per day to 45 minutes per day in the large city schools and 30 minutes per day to 47 minutes per day in the small school systems.
2. There is a marked difference in the amount of time spent in the lower grades (1st, 2nd, and 3rd). The lower the grade, the less the time devoted to arithmetic.
3. Very little difference is noted in the median time allotments in the upper elementary grades (4th, 5th and 6th).
4. A similar pattern in time allotment is observed for the small and large school systems. However, there is a difference of two to seven minutes between the medians of the small and large schools. This indicates that the small school systems spend more time on arithmetic than do the large city schools. This condition is especially true in the lower grades.
5. There is a wide variation of over sixty minutes in some cases between the time allotments within each grade level in the different schools. The lower the grade, the greater the variation.

Valid conclusions cannot be drawn unless certain differences in other educational systems are noted. In other nations of the world, the elementary curriculum may vary in length from three to eight years. The usual patterns for elementary education throughout the nations of the world are the four and six year requirements for elementary education. The four-year elementary cut off in many of these nations is the place where pupils are separated into groups for college preparation, for further technical training, or into the labor market after their compulsory education is completed.

Another difference is that most of the other school systems are on a five and one-half or six day week, as compared to our five days. Thus, it appears that more time is devoted to arithmetic in the other nations.

Many governments prepare a specific outline for education and suggest time allotments for arithmetic in each grade level. In some countries, such as Belgium and France, these requirements are part of the law. In other countries, such as Korea, officials recommend a specified program in mathematics. In England no definite time allotment is required but the children must pass a test at the end of their four-year elementary program.

In general, most nations which do permit control of their educational institutions by district or province, such as Germany or Denmark, do have a relatively similar program in mathematics throughout the nation. There are some exceptions, such as Canada, where noticeable differences in time allotment are evident between the provinces.

In summary, it is apparent that the majority of the educational authorities of the nations do provide some specific recommendations or a required curriculum in mathematics.

Time Outside the United States

The data for this study was obtained by several methods. Approximately sixty per cent of the information came directly from the educational authorities in the country. The other forty per cent was obtained by securing the information from a review of the recent literature (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) and by interviewing students and instructors from these countries. In all cases they were asked to give the daily time allotments for the first six years of their mathematics curriculum. In the cases where incomplete data was given for any of the six grades, the information was not included in this analysis.

The present table is based on the compilation of the data from thirty-two countries, not including the U.S.A. A breakdown by continents shows the following: 12 countries in Europe, 10 countries in Asia, 5 countries in South America, 3 in North America, 1 in Africa, and Australia. All contents have a fair representation, except for Africa where very little information was returned. However, considering the low level of education in this continent at the present time, this result could be expected.

The results of the daily time schedules for mathematics were tallied and the median for each grade level was determined. The per cent of nations which allotted a specified time for arithmetic was also computed for each grade level and is provided in intervals of ten minutes. The data was reported in the same manner as the previous article so that direct comparisons can be made.

Table I shows the time allotment for arithmetic for each grade level for the nations in this study.

TABLE I

TIME ALLOTMENT FOR THE ELEMENTARY GRADES IN OTHER NATIONS

	Median Minutes	Number of Mintues Per Day						
		0–9	10–19	20–29	30–39	40–49	50–59	60–*
Ist Grade	45		3%	6%	18%	37%	30%	6%
IInd Grade	45			9%	21%	35%	30%	5%
IIIrd Grade	47			3%	12%	41%	37%	6%
IVth Grade	47				9%	45%	40%	6%
Vth Grade	50				3%	48%	40%	9%
VIth Grade	50				3%	48%	40%	9%

* 1½ or 2 hours.

Table II

Time Allotment for the Elementary Grades in the Large Cities

	Median Minutes	Number of minutes per day						
		0–9	10–29	20–29	30–39	40–49	50–59	60–69
Ist Grade	23	14%	24%	39%	14%	6%	3%	
IInd Grade	32	3%	3%	38%	50%	3%	3%	
IIIrd Grade	40			3%	47%	29%	18%	3%
IVth Grade	45				3%	76%	15%	6%
Vth Grade	45				3%	70%	21%	6%
VIth Grade	45				6%	77%	15%	12%

Table III

Time Allotment for the Elementary Grades in the Small Cities

	Median Minutes	Number of minutes per day						
		0–9	10–19	20–29	30–39	40–49	50–59	60–69
Ist Grade	30	5%	9%	36%	25%	13%	7%	5%
IInd Grade	35			25%	46%	11%	9%	9%
IIIrd Grade	42			7%	36%	36%	10%	11%
IVth Grade	47			2%	11%	53%	21%	13%
Vth Grade	47				11%	57%	19%	13%
IVth Grade	47				11%	55%	23%	11%

Tables II and III (from the previous article) are included to show the average time allotments for the large city schools of the U.S.A. and the small schools, respectively.

Conclusions

The analysis of the data and consideration of the above tables produces the following information:

1. The other nations of the world devote more time to arithmetic than we do in the U.S.A.

2. Over 85 per cent of the nations in this study required the same length of time in the first six grades.

3. In the first three grades, the median differences were higher for the other nations as compared to the schools of the U.S.A. The differences shown were 27 to 7 minutes in the comparison with the large school systems and 15 to 5 minutes with the small city schools. There is a difference of only 3 to 5 minutes in the upper three grades.

4. There is much less variation in the ten minute intervals of time allotment in each of the lower three grades of the schools in this study, as compared with the U.S.A. Eighty-two per cent of the schools in this study used periods ranging in time from 40 minutes to 59 minutes, as compared with 23 to 42 minutes in the U. S. schools.

5. Since many schools are scheduled for a five and one-half to six day week, the total time allotted would be more than is shown by Table I. Thus, it should be pointed out

that these estimates in time allotments for the nations of the world are actually underestimates. This condition is particularly true in the Communist-dominated countries and the Far East.

6. Most of the schools which rank in the lower percentages of time allotment in mathematics are those which were influenced by the American educational system, e.g. Canada and New Zealand. The nations on the opposite extreme are India and Laos.

Most of the nations do provide more arithmetic instruction than we do. This answer the first of our proposed questions.

The question of the differences of time allotment is provided in the comparison of the first three years of arithmetic instruction in the elementary schools. The full period of arithmetic each day in the lower grade levels for the other nations is in marked contrast to the diverse time-allotments in the U. S.

The reason that less time is spent in arithmetic in the lower grades of our schools is due to certain philosophical and psychological concepts introduced in the last few decades. The "progressive educators" would have the children learn to be adjusted individuals and, thus, would tend to deemphasize mathematics since this topic is subject centered. The psychologists would wait until the pupils became more mathematically mature, i.e. they had the "readiness" to master these complex abstractions. Next, the Committee of Seven recommended the reduction of time in arithmetic in the elementary grades. Instructors were to introduce the topics at the age when the students could best master the topics in arithmetic.

Diversities in the lower grades of our schools can partly be explained by the great influence of these concepts. The advantage of this diversity is that it permits every school system to present its pupils with the system it deems best. However, a great disadvantage is apparent when students try to maintain their grades in the junior high schools and high schools in competition with those students who have had a greater preparation in the topics of mathematics.

Teachers who have taught in areas where students exhibit great differences in mathematical preparation have noted that this difference is difficult to overcome even with intensive study. In many instances students lose time in order to make up work that could have been learned earlier. While more information is needed on this topic, it appears that some uniform time-allotment should be agreed upon by the school systems in our nation to help reduce the difference in mathematical ability that exists at the present time.

The third question which was proposed, does increased time in arithmetic produce a difference in ability in students of different nations, can now be answered. During the Nineteenth Century, Germany was a weak confederation. However, after the Prussian consolidation which brought about compulsory education in mathematics and science, Germany made great strides in scientific contributions. Japan, a backward nation, until the arrival of Commodore Perry, grew to be a recognized power in the world in less than a century. These technological achievements were made because of the introduction of a new emphasis in science and mathematics. The advances made by Russia since the inauguration of a strong program in science and mathematics, has brought Russia in less than a half century from one of the weakest powers in the world to our present chief competitor. It should be noted that all of these countries made their great advances by means of the mass education of their people in science and mathematics. One needs to note that all of these nations make use of five to six periods of mathematics a week.

Whether we are in a race for "peaceful coexistence" or for supremacy of weapons, the need of our nation to make contributions in science and mathematics is vital.

Thus, from the analysis of this study the following recommendations will be made:

1. There should be a specified time allotment for the instruction of arithmetic agreed upon by the U. S. school systems to provide

a more uniform background for our pupils.

2. More arithmetic should be taught so that we may retain our present superiority in science and mathematics.

3. More studies should be made on the exact content of the arithmetic curriculum in other nations to find out the similarities and differences.

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EDITOR'S NOTE. We are grateful to Dr. Miller for assembling the data concerning the time spent on arithmetic in various countries. He has presented this in summary fashion in TABLE I which does not however identify the constituent countries. When the data for the United States is compared with that outside the country, if we should add a sixth session per week, that would mean a total of 331 minutes per week in the six grades in the non-United States group whereas the average in this country totals 239 minutes per week. This is nearly a 40% advantage. An additional factor not here accounted is the number of weeks spent in school each year which in many foreign countries significantly exceeds that in the United States. Perhaps it is surprising that our achievements are as good as they now are. As Professor Miller says, we have different views of education even within this country. He also points out that the nations that have made great strides in the past 100 years have had serious programs of mathematics and science and that if this nation wishes to advance it must take this work more seriously and provide adequate time for learning in these fields. In the April issue, Dr. Brownell reported that it was not uncommon for youngsters of age eight in Scotland to be spending an hour in mathematics per day. Of course their learning soon becomes more advanced than we have at this age. We too can do what is done in other countries if we wish but we must first re-examine our ideas of the purpose of education.

New York's Refresher Institutes

Beginning in the summer of 1958 the State Education Department of New York began sponsoring special institutes for elementary and secondary school teachers in the areas of mathematics and science. These were designed to enhance the knowledge backgrounds of these teachers. These institutes have continued throughout the state during summers and the regular school year. They are strategically located about the state so that most teachers have an opportunity to attend without traveling a great distance. Teachers receive the cost of tuition plus a small stipend to cover some of the expenses. Because so many institutes sponsored by the National Science Foundation are available to high school teachers, New York, has during the past year, concentrated on the elementary field. To date probably seven per cent of all of the elementary teachers of the state have availed themselves of these special institutes which are devoted to an understanding of the subject matter background which a teacher should have for the better service to her pupils. This is more of a study in depth and not a refresher in the subject matter of the elementary school. The program has met with a great deal of success and particularly when the institute has been conducted by one who is familar not only with the new developments in mathematics but also with the clientele. This is one of the most comprehensive inservice programs in the country. It will be continued for another year.

Number, Numeral, and Operation*

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Number

NUMBER IS AN IDEA, an abstraction. Collections or groups or sets are said to be equal in number when the members of the collections can be arranged in one-to-one correspondence. The collections shown obviously are equal in *number*. The quality which they (these collections or groups) have in common is called *fourness*.

x	*x*	*x*	*x*
0	0	0	0
h	*h*	*h*	*h*

The collections shown below are unequal in number. The number of X's is smaller than the number of O's; the number of □'s is larger than the number of O's. The common or standard names of the numbers of objects represented above are four, five, and six. In our culture the names of the "natural numbers," in order, are *one*, *two*, *three*, *four*, etc.

X	*X*	*X*	*X*		
0	0	0	0	0	
□	□	□	□	□	□

We can not write, or see, numbers. They are, as we said, abstractions. However, we can represent the number in a collection by pebbles, marks, repeated sounds, pictures, words, and by symbols called *numerals*.

Numerals

A numeral is a symbol which, by agreement, represents a number. In the Roman system of numerals the first ten consecutive natural numbers (counting numbers) are represented by the numerals I, II, III, IV, V, VI, VII, VIII, IX, and X. In our system of numerals, the Hindu-Arabic system, the first ten consecutive natural numbers (counting numbers) are represented by the numerals 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Thus the number expressions 10 and X are equivalent —they represent the same number, to which we give the name *ten*.

Digits. The ten number symbols 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0 are often called *digits*, or *figures*.

A system of numerals using ten digits is called a *decimal* system. A system using only five digits, 1, 2, 3, 4, and 0, is a *quinary* system.

Base. In counting, when we reach a specified number, the *base*, we start over again, thinking, base and one, base and two, base and three, etc. The base is a two-digit numeral, written as "10." The "10" is interpreted as "one base, zero ones." "Base and two" is written as "12" and in a decimal system means ten and two. "Two base" is written as "20," meaning "two tens and zero ones," or "twenty."

The numeral "12" in base five means "1 five, 2 ones." Thus "12" in base five stands for (5 and 2) or "7" in base ten. To interpret a numeral we must ascertain the base upon which the numeral system is built.

Positional value. The position of a digit in a numeral determines "its value,"—the number it represents. In the numeral "32" the digit 3 (in base 10) stands for or "has a value of" 3×10, and the digit 2 has a value of 2×1.

* Digest of an address at the February, 1959 meeting of the New Jersey Association of Teachers of Mathematics.

This paper deals with the writer's notion of the nature of arithmetic,—with the concepts and principles which determine the meaning and rationale of arithmetic.

Powers of base. We often say that our system of numerals is a polynomial consisting of a series of powers of 10 (the base.) The polynomial $(10)^3+(10)^2+(10)^1+(10)^0+(10)^{-1}+(10)^{-2}+(10)^{-3}$ is equivalent to $1000+100+10+1+.1+.01+.001$, or 1111.111, as shown in chart.

$(10)^3$ =	1000
$(10)^2$ =	100
$(10)^1$ =	10
$(10)^0$ =	1
$(10)^{-1}$ =	.1
$(10)^{-2}$ =	.01
$(10)^{-3}$ =	.001
	1111.111

Similarly, 234.56 means $2\times(10)^2+3\times(10)^1+4\times(10)^0+5\times(10)^{-1}+6\times(10)^{-2}$. Note that the negative powers of the base give meaning to tenths, hundredths, thousandths.

Obviously, the digit and its position in a numeral indicate the number it symbolizes.

Should we distinguish, in the elementary school, between number and numeral? To focus our thinking on this issue, let us consider the problem of finding how many tomato plants are needed for 8 rows, with 12 plants in a row. To compute the product of the number of rows and the number of plants in a row, we work with numerals, as shown. Then our thinking if we distinguish between number and numeral, becomes

$$\begin{array}{r} 12 \\ \times\ 8 \\ \hline 96 \end{array}$$

The product of the numbers represented by the numerals 12 and 8 is the number represented by the numeral 96. Hence the number of plants is "ninety-six."

This language does not facilitate communication, even though it is essential in maintaining the distinction between number and numeral. We add, subtract, multiply and divide numbers, not numerals.

To observe the distinction between number and numeral in the discussion of fractions leads to even greater blocks to communication. As a number, a fraction is a quotient of two numbers; as a numeral, the fraction numeral for a number half way between zero and one is the equivalence set: 1/2, 2/4, 3/6, 4/8, 5/10, etc. Again, we can add, subtract, multiply or divide the numbers represented by fraction numerals.

The Report of the Commission on Mathematics advises that we should use the one term *number* for both meanings. Also the School Mathematics Study Group states that it may be cumbersome to the point of annoyance to speak of *adding the numbers represented by the numerals.*

Operation

Having considered very briefly the *number* and *numeral* part of our paper, we now turn to a consideration of *operation.*

Addition. On this number line the length of segment OA is 5 units, and the length of

AB is 8 units. To find the length of OB, the sum of the lengths OA and AB, we employ the operation of addition. The sum of 5 and 8 (the addends), as can be seen by moving from left to right, is 13.

Subtraction. If, on this number line, OB is 13, and AB is 8, then OA is $OB-AB$, or $13-8$, or 5. Thus if $5+8=13$, then

$13-8=5$. Subtraction then is the inverse (opposite) of addition. In general, if $a+b=c$ then $a=c-b$, and $b=c-a$. We say that subtraction is defined as the inverse of addition. Thus, any "addend plus addend equals sum" statement implies a "minuend minus subtrahend equals difference" statement.

Multiplication. We agree that "8 times 5" or "8×5" means the sum of 8 fives, or 40. In the vocabulary of multiplication, the 8 is the multiplier, 5 is the multiplicand, and 40

is the product. Symbolically: $a \times b$ means $b+b+b+\cdots$ (to a addends). When it is unnecessary to distinguish between multiplier and multiplicand, we call them factors. Using a and b as factors, and c as their product, the basic multiplication relationship is symbolized as $a \times b = c$.

Division. The operation of division is the inverse of the operation of multiplication. Thus, in division, either of the two factors of a product is equal to the product divided by the other factor. In division, then, we are concerned with finding a missing factor (the quotient) when the product of the two factors (the dividend) and one factor (the divisor) are known.

The terms *partition division* and *comparison division* are used to designate the two different uses of the operation of division,—a distinction needed in problem solving. The division algorisms,

$$3\overset{\displaystyle 4}{\overline{)12}}, \quad \frac{12}{3}=4, \quad \text{and} \quad 12 \div 3 = 4,$$

may mean (1) dividing 12 into 3 equal parts (partition), or (2) comparing (measuring) the 12, using 3 as a unit of measure. The second meaning is often called ratio.

Laws Governing Operations

Arithmetic computation is governed by and employs three mathematical principles, usually called laws: the commutative law, the associative law, and the distributive law.

The commutative law of addition and multiplication declares that

$$a+b=b+a, \quad \text{and} \quad a \times b = b \times a.$$

The associative law of addition and multiplication declares that

$$a+b+c=(a+b)+c=a+(b+c),$$

and

$$a \times b \times c = (a \times b) \times c = a \times (b \times c)$$

The distributive law of multiplication declares that

$$a \times (b+c) = (a \times b) + (a \times c).$$

Since

$$a \div b = a \times \frac{1}{b},$$

(the dividend $\times$ the reciprocal of the divisor) the above principle implies that

$$(a+b+c) \div d = \left(a \times \frac{1}{d}\right) + \left(b \times \frac{1}{d}\right) + \left(c \times \frac{1}{d}\right).$$

Throughout the elementary school and in algebra, these laws underlie the processing of numerals. To illustrate, the number expressions

$$\frac{15}{3}, \quad \frac{1}{2} \text{ of } 10, \quad \frac{18-3}{3}, \quad 2+3, \quad \text{etc.},$$

may be transformed into or replaced by the equivalent 5.

Using one or more of these laws, we simplify number expressions by performing the indicated *operations.*

Problem solving. Our great concern about operation (computation) in arithmetic is that it is the final step in problem solving. Without intelligent computation we are unable "to find the answers" to our problems. In this frame of reference, let us consider the use of the *equation,* a symbolization of the elements (concepts) inherent in a problem. Some allege that the equation is the magic key in problem solving.

To illustrate, consider the problem: Jane has 18 cents, and wants to buy a pen which costs 25 cents. How much more money does she need to buy the pen? Here we have three elements or concepts, the amount Jane has, the amount she still needs to get, and the cost of the pen. These are related. Interpretation or analysis of the problem suggest the relationship. Verbally stated, the relationship is: *Amount she has, plus the amount needed, equals the cost.*

This relationship may be more symbolically stated in equation form, $18+N=25$.

From here, however, the third grader yet has a long way to go to solve the problem.

Either the equation or the verbal statement of the basic relationship, is only a step toward the solution,—a strategic step though it be.

The thinking leading from the equation to the solution is dependent upon a fundamental principle (not an axiom of algebra) which is a prerequisite to the solution. Without the use (conscious or intuitive) of this fundamental or key principle, the pupil could not be expected to think.

$$\begin{array}{r} 25 \\ -18 \\ \hline 7 \end{array}$$

Obviously the secret to the solution of the equation is: An unknown addend equals the known sum of the two addends minus the known addend. Hence, if $18 + N = 25$, then $N = 25 - 18$, or 7.

Now it is not the equation, or the verbal statement, which brings forth the solution. It is this basic mathematical relationship between addition and subtraction, not the equation, which brings forth the solution.

To think mathematically about the solution of this problem, the pupil does not revert to the problem setting. He proceeds from the $18 + N = 25$ to $N = 25 - 18$ by this principle of inverse relationship.

A second illustration will be helpful. At 5 cents each, how many pencils can be bought for 60 cents? Here again we have three elements or concepts: the price per pencil, the number of pencils being bought, and the total cost of the pencils. These are related. Expressed in a verbal statement, the relationship is "Number of pencils times 5 cents equals 60 cents." Expressed or translated into an equation, the relationship is $N \times 5$ cents $= 60$ cents.

Again, the pupil is far from the solution. His thinking must be the equivalent of "Here are two factors, N and 5, and their product 60." He must recall the fundamental principle of inverse relationship between multiplication and division: in this case, the unknown factor N equals the known product 60, divided by the known factor 5. Now he senses the operation which leads to the solution.

We have considered the thinking which, beginning with an analysis of the problem, results in relating its three elements (concepts), giving either the verbal statement, or the equation, $N \times 5 = 60$. The next all important step, we repeat, proceeding from here and leading to the choice of operation needed to obtain the answer, is the principle "either of the two factors equals their product divided by the other factor," or in the language of the rule: To find an unknown factor when one factor and the product of the factors are known, divide the known product by the known factor. This reasoning transforms the original equation $N \times 5 = 60$ into the equation $N = 60/5$.

The equation $N \times 5 = 60$, obtained by analyzing the problem situation, does not *per se* indicate the operation to be used to obtain the answer. From it, the pupil must see, by using the inverse relationship between multiplication and division, that $N = 60/5$. This *seeing*, this *reasoning*, from $N \times 5 = 60$ to $N = 60/5$ is the magic key to choosing the operation which leads to the solution.

Pedagogical Implications

This paper has given major consideration to the structure of arithmetic, to the nature of arithmetic. No consideration has been given to "how to teach arithmetic." The writer agrees with those who assert that we who teach arithmetic profit greatly by deepening our understanding of the nature of arithmetic. Fortunately we soon shall have a Year Book by the National Council of Teachers of Mathematics, edited by our esteemed colleague Doctor Grossnickle, devoted both to the nature of arithmetic and to the teaching of arithmetic. Careful study of this Year Book will surely be rewarding to us.

(*Editor's Note on page 230*)

The Slow Can Learn

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The old demand used to be that we keep up with the Joneses; now it has changed—at least in education—we must keep up with the Sputniks. Where formerly an expenditure of $50 for supplies for mathematics was considered a needless extravagance, millions are now offered eagerly. At last the mathematics teachers have come into their own. With these opportunities, teachers of mathematics must coolly evaluate the situation and plan wisely for the best education for all our youth.

The assumption of the lay public frequently seems to be that, with large doses of mathematics (especially the so-called modern variety) all children can become gifted mathematicians, leaders against aggression in these threatening times. We wish it could be true, but firing the Sputnik did not change human mental capacity; we still have the talented, the average, and the less able learners. The man on the street does not realize that the gifted children comprise only the top 1% to 10% of the school population, that without followers there can be no leaders, that for effective functioning for these followers as well as for the leaders, there must be education suited to their needs and abilities.

Numerous well-publicized and heavily endowed studies are in progress attempting to find what changes should be made in the mathematics curriculum for talented youngsters. These experiments are usually planned to discover possible attainments of the gifted on the secondary school level and are laying the ground work for college majors in mathematics, science and engineering. No doubt eventually these investigations will reveal some very important information about the best topics to teach and the best methods to use for these potential leaders.

Naturally relatively little attention has been paid recently to new methods or materials for training the aforementioned "followers" who are the average and the slow learners. The latter were studied as a major problem in the pre-Sputnik days. The Guidance Pamphlet of the Council (revised in 1953) gives a check list of 29 competencies in mathematics necessary for personal use and for everyday occupations. Many workers in education have assumed that this check list may be used as a basis of mathematical training for both the average and the slow learners. The average youngsters should receive as heavy an enrichment in algebra and geometry as their abilities permit, and the slow learners would complete the basic work at their optimum rate of speed perhaps needing the entire twelve grades for a satisfactory mastery of the topics given.

The Role of Arithmetic

Since arithmetic is the most important item on the list and is the basis for all mathematics, shall we examine briefly some of the problems that teaching it presents?

The men who pay the school taxes expect the products of those institutions to be proficient in the 3 R's. Without a working knowledge of the last R, how can people figure their wages, run their households, prepare financially for the future? In spite of this emphasis, income tax directors report that most of the errors in income tax returns are due to mistakes in arithmetical computation. In all mathematics the right answer is still important.

The occasional statement that all computation nowadays is done by machine is a bit optimistic. Machines do not and cannot

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replace all computation by human beings. Few homes are equipped with computing machines and only large industrial and business houses can afford them. Small businesses must rely upon the traditional methods. Even when computing machines are used, they depend for their effectiveness upon the mathematical ability of their operators. The outlook seems to be that with increased precision and automation by machines and with world trade competition there will be an increased need for more and better computation by men.

Although arithmetic is accepted as a needed subject, its difficulty is seldom recognized. It is assumed that any material taught in the elementary school is easy. People do not remember the ages-long struggle of the greatest minds of their times to build the science as we know it; that creating a notation for fractions was a major invention developed over the centuries; that writing fractions as decimals and computing with them is a relatively recent accomplishment. Of course parts of arithmetic are easy and parts are difficult. The geometric shape of the circle is easier for the kindergarten child to comprehend than the simplest number combination. The informal solution of an easy equation requires far less mental effort than dividing by a two-figure number. The $3\frac{1}{2}\%$ or $3\frac{3}{4}\%$ of wages required as a social security tax that must be paid and may be understood and figured by most workers, presumes a considerable knowledge and skill in many topics of arithmetic. Yet we expect children to master this science in a relatively short time.

Since arithmetic is the backbone of the mathematics courses for the non-academic student in grades one through twelve, let us examine the training his teacher had received. Formerly it was assumed that studying arithmetic as a pupil in the elementary school was ample preparation for teaching the subject. It is true that schools of education have always offered, but not required or at times advised, methods courses in arithmetic, but the picture is brightening. Some colleges are now requiring that candidates for an elementary school diploma elect such courses; some schools have added to this requirement a course in the subject matter of arithmetic—or at least a certificate showing that the candidate has passed with credit a standardized test in arithmetic. Yet to acquire a thorough preparation in teaching arithmetic sometimes works a hardship on the elementary school teacher who must be a specialist in half a dozen subjects—another argument in favor of departmental teaching.

Few instructors of the non-academic mathematics in the higher grades have taken any college courses in arithmetic and sometimes they are not trained in mathematics but are given a class in general mathematics as an extra assignment.

To correct this unhappy situation there has been in-service training of teachers, some being done locally, some in extension classes of nearby colleges and summer school courses have been planned to help teachers of experience. A few summer workshops or institutes have been devoted to the better preparation of teachers of arithmetic. It is to be hoped that these will increase in number and in influence and that they will include in their studies a thorough examination of the structure and subject matter of arithmetic including its history, that they will review the important research of arithmetic, that they will study methods and devote a period of time to the problem of arithmetic and the slow learner.

Sectioning and Grouping

For many years experiments were made with ability grouping at various levels, but many administrators rejected this method of organization on the grounds that it was undemocratic and did not produce superior results. Probably the real reasons for the failure were due (1) to the opposition of the teachers who were not prepared to teach the slow learners and did not wish to do so; (2) to the fact that parents were not educated to understand the opportunities being offered to their children; and (3) to the peculiar, but prevalent ideas that the less able

children were expected to be taught the same subject matter, by the same methods, and in the same length of time to reach the same standards set for the gifted children of the grade. These difficulties are now being overcome and ability grouping is being generally accepted as an effective way of meeting the problems of educating all types of children to capacity and to meet their future needs.

At what grade level this ability grouping should begin has not been scientifically determined. It is agreed that, as children developed, the divergence between the slow learner and the talented increases rapidly. Perhaps the variation in abilities shown by the youngsters in a particular school provides the answer to the question of the time at which this grouping is desirable. Experiments have shown that in some localities it is advantageous to divide the pupils in the first grade; other school systems find a division in the third grade efficient; still others feel postponement to a time not later than the seventh grade is most effective.

It is not always easy to section children according to ability even in a large elementary school, but fortunately if a child is slow in one subject he is usually slow to learn in all of them. If the school is small, ability grouping requires more imagination. Some teachers have tried dividing a class into sections—a device used with great success by teachers of reading. If the self-contained classroom has given way to departmental teaching, ability grouping is not too difficult to administer.

The Role of the Teacher

The teacher is the most important factor in producing success or failure in educating the less able children. How can the reluctance of some teachers to accept such an assignment be overcome? The administration should speak of this as a special appointment to be given to superior teachers. It has been necessary in some places to give additional increments to the teachers of the slow learners. In an occasional school system the attitude of the other faculty members has been to discredit the instructor of the slow learner. There should be education so that this condescension is changed to respect. The fact should be emphasized that although the subject matter for these children is simpler, the teaching of it is far more difficult and is based upon a thorough understanding of a special psychology.

Suppose we could order from an educational supply house a perfect teacher for the less able children. What specifications should we write? In addition to the usual qualifications for a successful instructor, the ideal mentor for the slow learners should have an unusually deep love for the young of the human family, so strong a feeling that it is sensed by her charges. A child will do almost anything for a person whom he thinks really cares for him. She must have the patience of a long-suffering Job and an equal amount of tolerance and sympathy. But, combined with these characteristics, she must be a firm but tactful disciplinarian. She must foreget her college-fostered vocabulary and learn to speak simply but correctly in the language of her pupils. Although she should have a complete mastery of the subject matter to be taught, she should not be too academically minded but should have the imagination and information to point out the applications of arithmetic to its myriad uses in the world.

When a teacher unusually successful with slow learners is found, some administrators are tempted to give her such an assignment year after year. In the long-range picture, however, this is not desirable. It is good in-service training for all teachers to have at least a short experience in teaching the less able; if a teacher always teaches the slow learners, the children she teaches may automatically be labelled "dummies" by their unkind playmates; also, the teacher herself may become so accustomed to inferior performance that she forgets high grade standards.

Having selected the slow learners and chosen their teacher and appropriate subject matter, the next question to be met is how are these less able children to be taught?

The classes should be small, not more than 25 can be efficiently handled. The slow learner requires much individual attention because he is unable to isolate and solve his own difficulties. In addition there are usually a larger per cent of reluctant learners and discipline cases that take extra time and effort from the teacher.

The wise instructor will as soon as possible re-examine any questionable cases to see if they have been correctly classified. Into the class there may have strayed a reluctant but bright learner and some children whose apparent dullness is due to physical and not mental causes. When the new eye glasses, hearing aids, better clothes, more nutritious food have made their correction and the reluctant learner has been convinced of his potential powers, the children benefited should be transferred to the proper section.

Meanwhile the teacher should convince herself that although she is teaching slow learners, they are *learners*, if slow, and can be taught. Thoroughly convinced of this fact, she builds up the ego of her charges by her attitude and by assigning various simple tasks at which even the slowest can succeed with ease. She proceeds at a very slow pace because her charges are correctly named not only as learners, but *slow* learners. Each topic is broken down into tiny steps, and the material is developed omitting none of these small gradations of difficulty which the slow minds must surmount.

The time honored cycle of motivation, understanding, drill and application is as valuable a learning sequence for the less able as for the brighter youngsters.

When children are very young, they are eager to learn as everyone knows who has attempted to answer all the *what's* and *why's* of a little child. As they grow older this lively curiosity dims and they want to know why they have to learn facts and acquire skills. Here the wise teacher uses motivation but being careful to base her incentive upon things vital to the child rather than of interest to the adult. To amend an old saw, even though you lead a horse to water and can't make him drink, you can feed him salt.

The very successful program devised for teaching the meaning of number proceeding from the concrete through two stages of semi-concrete to the abstract can be adapted with profit to the introduction of many other new topics. The skillful teacher makes use of many concrete materials; especially helpful are those that can be touched, moved about, and are brightly colored. Talented tots enjoy abstractions, they may sometimes omit the concrete phase of learning altogether, but the less able find abstractions difficult and must be led to them via the concrete route.

Children of lesser ability enjoy practice; it does not bore them. They like to succeed (who doesn't?), and they can succeed through drill.

However, their power of transfer is slight so usually the teacher will need to point out the transfer to them.

Their ability to make discoveries is definitely limited to very simple cases; try as they may they usually have to be told.

Since their ability to forget is much superior to their ability to remember, the teacher of the slow learners finds herself committed to a continuing program of "review and enlarge." With each repetition there is a small gain in the amount of material permanently retained. More learning —and happier learning—takes place if the slow children are confronted with a large number of simple problems to solve instead of a few that are more difficult. But through great effort of both the learner and the teacher a surprisingly large and useful store of basic information and skills may be acquired.

The attention span of the dull-normal is short; probably twenty minutes is the limit of the time they may spend profitably on one type of work. It is then wise to change the bodily as well as mental activity.

A story is told of a father who had never studied algebra but who was a great help to his student son. When asked by the boy's teacher how he performed this miracle he explained, "When Jim asks me how to do a problem, I always say, 'Read it again'!" The father was wise and his method may be

copied by many a teacher. A learner, even a slow learner, can be taught to read an arithmetic textbook and profit by careful reading and rereading; much may be discovered about the amount he comprehends by observing his intonations as he reads aloud; he may learn to understand, use and spell the correct mathematical words and phrases by repeatedly reading them.

Emotional appeal has always appeared first in the textbook of tricks of politicians, those masters of practical psychology. An emotional aspect—happy if possible—adds magic to the learning process. Praise, deserved and honestly bestowed, is probably the most powerful tool that a teacher has at her disposal.

Teaching the less able children is an adventure beset with hard work, discouragements, failures, unforeseen difficulties and surprising successes. For encouragement one needs merely to observe the happiness and gratitude of those youngsters who have gained a measure of confidence and self-respect, who have found they are able to learn and realize they have made some advance in knowledge and skills during the year, who cherish with pride their newly acquired powers and no longer have the attitude of indifference and fear of failure.

EDITOR'S NOTE. From her many years of experience, Miss Potter knows that "The Slow Can Learn." While she has not specifically defined the level of these pupils, we can all recognize them. Frequently we are apt to include in the group a child who seemingly is slow because he has other disturbances. Hence we must be willing to hold our grouping flexible. Patience and humility coupled with understanding arithmetic and the child are desired in all levels of teaching but are particularly important with our slow learners. What about the subject matter? Miss Potter plainly states that we should not expect the slow to travel the same route and to the same destination that we have set for the average and better pupils. She points out how it is better to succeed at several small tasks than to be frustrated by one larger problem. The slow can think and draw conclusions but they need more time and their routes are not as direct and occasionally they get lost when they fail to note a "signpost" which is clearly evident to their superiors. But we can help them and we must.

Number, Numeral, and Operation

(*Concluded from page 225*)

EDITOR'S NOTE. If we are to be precise according to Dr. Clark we may speak of adding and multiplying "numbers" but this does not mean that it is the symbols or numerals that are being added and multiplied. How important is it for a teacher and a pupil to realize that *number* is an abstraction in the human mind? This level of sophistication should be held by the teacher but she should not confuse children by insisting that all their statements exhibit an adult level of discrimination. We must allow for and even encourage a maturation in concepts and in the language of pupils. That is the function of education. How important is it for a child to know that he is using the commutative or the associative law? How were these "laws" established? Are they mere agreements by mathematicians or are they inherent in the nature of number systems? They are "minimum consistent agreements" which apply to our number system. It is not important that they be named in the elementary school but the principles of these laws are very useful. For example, the distributive law explains that $5\times72=5\times70+5\times2$ or $5\times2+5\times70$ and this is not always apparent in our usual algorithm. And of course the associative law is involved in column addition. Dr. Clark's discussion of problem solving places stress upon discovering the mathematical relationships within the problem and then thinking with these toward a solution. Reasoning is important as a process and as a habit. Let us encourage thinking and reasoning.

THE ARITHMETIC TEACHER

This is the last issue to be prepared and edited by Ben A. Sueltz, editor, and John R. Clark, Marguerite Brydegaard, E. Glenadine Gibb, and Joseph J. Urbancek, associate editors. The journal now prints more than 13,000 copies each issue. When Mr. Sueltz was asked to establish the journal seven years ago he hoped to reach a circulation of 10,000 in five years. He is very pleased with the status the magazine has achieved. Now he is happy to transfer the obligation to Dr. Gibb who will carry the service of the National Council to teachers of arithmetic through the pages of the journal. All correspondence and manuscripts should be sent to her at Iowa State Teachers College, Cedar Falls, Iowa.

The Relationship of Socio-Economic Factors and Achievement in Arithmetic

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VERY FEW STUDIES which examine the relationship between arithmetic achievement and the socio-economic conditions of the achiever's family have been published. If more was known in this area, guidance counsellors and those people responsible for placement of school children in a particular program or class would be better equipped to perform such placement. A study of this nature can be helpful to the teacher of arithmetic, as well.

The purpose of this study, undertaken in the eighth grade of North High School in Valley Stream, Long Island, New York, is to determine to what extent there is a relationship between a child's achievement in mathematics and the socio-economic conditions of the family. The study attempts to clarify the question of a causal relationship between home environment or, at least, some aspects of home environment, and arithmetic achievement. An arithmetic achievement test is used as a measure of achievement in this study. A questionnaire is used to establish the socio-economic factors. This study also attempts to determine, if possible, why some groups which are assumed to be equal in ability perform better in mathematics than others. In other words, to what extent do the social and economic factors which account for success among some and failure among others determine achievement as well as the native ability or aptitude of a person.

There are certain limitations to the study. The sample used was taken entirely from one high school located in a suburb of a large metropolis. Thus, the study excludes other geographic groups which are representative of American society. The study is further limited in that the sample used was of school children in mainly the middle income group and lacking in the extremes on the socio-economic scale. The study group was made up entirely of white children.

Because of the above limitations this may be considered a pilot study in the sense that any conclusions drawn only point the way to a need for further research in the areas of arithmetic achievement related to socio-economic conditions. Nevertheless, the conclusions drawn are valid for the group studied and they are significant in pointing the direction for further research.

Design of Study

The study followed two patterns, that of the survey and also that of the *ex post facto* design. The following three unpublished primary sources were used: (1) a questionnaire (2) results of the Mooney Check list and (3) an arithmetic achievement test.

The study uses, mainly, the achievement test scores and the questionnaire to determine a relationship between socio-economic factors and arithmetic achievement. The questionnaire was administered to 208 eighth grade pupils in North High School. The arithmetic achievement test was given to the eighth grade on three occasions; first in September, 1958, and again in January, 1959, and finally in March, 1959.[1]

As an aid for guidance, the Mooney Check list was administered on October 9, 1958 to all eighth graders to learn what were the common concerns of the student. No identification was put on the sheets in order that most valid and useable results could be obtained. The Mooney Check list is used mainly as a check on the validity of con-

[1] The pupils in the seven classes tested were given thirty minutes to do the test. Those finishing before this time were instructed to recheck their work.

clusions reached in this study when a positive or negative relationship between a socio-economic factor and achievement in arithmetic is apparent.

A further check for validity is in the achievement test itself. Administering the test to the group three times shows not only what they achieved through seven months of learning process but also is a guarantee of the validity of the test results.

The plan of the study is this. *First*, the grouping by the guidance counsellor into three groups; "enriched," "average," and "slow," is accepted as being valid. Because of the many variables involved in the grouping, it is impossible to check on the validity of the guidance counsellor's grouping. However, as part of this study, the class mean on the arithmetic achievement tests is related to the grouping.[2] *Second*, the socio-economic status as arrived at from the variable is compared with the arithmetic achievement.

The tables were made in two ways: (1) A frequency distribution to show a comparison of the achievement of the seven classes and the ranges of the scores, and (2) tables showing the correlation between a particular socio-economic variable and the arithmetic achievement expressed in quartiles. The four quartiles are used to show the relationship of those scoring in the upper quarter, above the median, below the median and in the lowest quarter.

The conclusions of the results of tabulating the test scores and questionnaires follows. In addition to an analysis of results, certain class distinctions were developed based on the questionnaires. From these general distinctions a profile or composite was constructed for each class.

Class Profiles

The following profiles were developed from the relationships seen between the results of the questionnaire and arithmetic achievement and also as a result of analysis of the Mooney Check list by class. From this data a representative child was constructed for each category.

Profile of an "enriched child"

The child in the enriched classes may be characterized by the term achievement. He is in the enriched class because of his over-all achievement in school plus achievement in the other significant areas, namely; in general intelligence and in his family's social and economic background.

On the arithmetic achievement test, his scores ranged from 10 at the bottom to a perfect paper. The mean on the first testing was thirteen in two of the classes and sixteen in the third class. The class means varied between 17 and 21 on the second testing and between 21 and 23 on the third testing. In other words, arithmetic achievement is high.

The enriched child also achieves in after school activities. Some of these activities are: Scouts and Hi Y's, sports, reading, playing a musical instrument, and watching television. The child in the enriched class has the largest number of after school interests.

There is no general pattern for church attendance. It varies from regular to irregular.

The child's parents are also successful. Typical occupations for his father are teacher, accountant, owner of a small business or manager of a branch office of a large business. Both of his parents live with him and both parents were born in New York City or adjacent areas. His parents own their own home which has from six to nine rooms. The family usually reads the *Herald Tribune*, *New York Times*, and the local county papers. They give their *children* a large allowance, usually $1.50 or more a week.

Because of relatively stable economic conditions, there is no concern about money expressed by the child. He does express a good deal of anxiety in two areas, however; concern over examinations and grades, and in being popular.

[2] Because only seven classes had taken the test in September, 1958, the eighth is eliminated from the study.

Profile of the "average child"

The child in the average classes presents a varied background. On the arithmetic achievement tests, his scores ranged from seven to twenty-five. The mean average of the second testing was 14 to 16 and on the third testing 16 to 18. The arithmetic achievement corresponds to other school achievement which is in the middle of the grade.

The average child has a few interests after school. He spends this time in sports and in watching television. His church attendance is regular.

The amount of anxiety that the average child has varies but is generally very little. Areas of concern are in choosing subjects next term, lack of interest in some subjects, restlessness in class and desire to improve his appearance.

Both of his parents are living at home. Typical parents were born in various places, often New York City but also other parts of the United States or some city in Europe. They own a six or seven room one-family house. His father is usually in some form of skilled work such as a mechanic or electrician, or he owns a small shop, a barber shop or photography store. The family newspapers are of the middle or low middle cultural level, the local papers, *Newsday* and *Long Island Press* and the *Daily News* or *New York Post*. They give *him* an allowance of about $1.50 a week.

Profile of the "slow average child"

The slow average child is termed so because of his below average overall achievement. His arithmetic achievement is the lowest with a mean average of 12 on the second and third testings and the range varying from a low of 3 to a high of twenty. No improvement is seen between testings.

Both of his parents live at home in their six or seven room one-family dwelling. His father and his mother were both born in New York City, moving to the suburbs during the last five years.

The slow average child's father is either a skilled or semi-skilled worker or owns a small business, e.g., a candy store or a sign painting concern. The family reads the local newspapers and also the *Daily News* and *Daily Mirror*. They give each child an allowance of about $1.50 a week.

Church attendance is regular.

The slow average child has few interests. He participates in sports but does little else.

The only area of concern that the slow average child has is his inability to get along with his teachers.

Summary and Conclusion

Guidance counsellors vary in the emphasis placed on five criteria, i.e., (1) health, (2) intelligence, (3) achievement, (4) social factors, (5) scholastic record. Some use social factors to a large extent, where others depend almost entirely on intelligence test scores. For the 8th grade classes studied, the Otis test scores and the 6th grade marks in citizenship education and English were the bases for grouping. It was assumed that if the scholastic record in mathematics is ignored in grouping, as was the case for the classes now under study, the child's achieving to his level may be thwarted. However, certain elements in his home environment may compensate for this lack of concern with the mathematics achievement of the child. When the child does not achieve at his level in arithmetic, there is a need for a new criterion for grouping, a criterion which would consider mathematics as well as citizenship education and English.

As a result of this study, implications result for the method of grouping that is being employed. The study shows that the grouping is in a general way accurate enough for arithmetic. Since the attempt at grouping was to group for citizenship education and English, this is surprising. The skills do not necessarily go together.[3]

[3] Current research has shown that when homogeneous grouping is employed, the data seems to indicate that success depends on the areas of expected achievement. However, the evidence is not conclusive. Kenneth E. Brown, "Research in Teaching High School Mathematics," *The Mathematics Teacher*, December, 1958, p. 594.

TABLE I

ITEMS USED IN CLASSIFYING STUDENTS*

Item	Number of checks in class number						
	1	2	3	4	5	6	8
Not spending enough time in study	2	12	11	8	8	6	13
Frequent headaches	1	6	1	10	1	3	5
Wanting more time to myself	5	7	2	9	7	4	9
Daydreaming	4	8	14	9	4	7	9
Not living with my parents	1	0	0	0	1	0	0
Parents separated or divorced	1	0	0	1	0	1	2
Not having any fun with mother or dad	3	5	3	3	4	2	6
So often feel restless in classes	4	12	7	7	6	10	7
Parents favoring a brother or sister	2	5	5	4	4	2	7
Not interested in some subjects	5	11	8	10	10	6	9
Not getting along with a teacher	11	4	6	4	5	5	4
Living too far from school	2	6	4	11	6	5	10
Unhappy too much of the time	3	2	1	3	1	0	0
Being tempted to cheat in classes	5	8	5	7	3	5	2
Trouble with mathematics	5	9	14	6	9	1	10
Parents working too hard	1	4	7	5	22	4	0
Don't like to study	4	6	8	6	8	5	7
Poor Memory	1	7	3	5	3	2	9
Worrying about grades	5	7	12	11	7	5	16
Worrying about examinations	4	10	12	17	9	8	18
Getting low grades	4	1	8	6	2	2	9
Afraid of failing in school work	4	6	8	13	4	4	13

* These items were selected from the 330 on the Mooney check list.

This points to a need for further research to determine if the child who is not achieving in arithmetic is not achieving in the other subjects. If he is only a low achiever in arithmetic, it indicates a faulty criteria for grouping. If he is generally low in achieving, the grouping is adequate but the child is lacking in some way in order to achieve in arithmetic.

There are also indications of value of drill and review for the child. The results of the study are quite conclusive that there is real value in reviewing previous material for the "average" and "above-average" child. The "slow average" child appears to gain little from this review. This suggests the need for further research, as only one "slow average" class was included in this study.

Regarding the relationship between socio-economic conditions and arithmetic achievement, it was found that a positive relationship is present between some of these social and economic conditions and a child's achievement in arithmetic. The relationship is not present between the status elements but rather between cultural aspects of the home and arithmetic achievement.[4] For example; there is a positive relationship between more intellectual newspapers (i.e. *New York Times*) being read in the home and the child achieving in arithmetic and also between the less intellectual and picture type paper (i.e. New York *Daily News*) and a lack of achievement. A positive relationship also presents itself between a parent's occupation in a profession and the child's achievement. This relationship is interpreted to be a cause of achievement because of what the child received from his parent and the attitude of his parent. These relationships point toward an intellectual achievement on the part of the parents. If they

[4] The results of the most recent report on this relationship between economic status and arithmetic achievement are inconclusive. See Leland H. Erickson, "Certain Ability Factors and Their Effect on Arithmetic Achievement," THE ARITHMETIC TEACHER, V, 6, December, 1958. Pp. 292–293.

have been successful in their schooling they would tend to influence their children towards achieving a similar success.

The results of the Mooney Check list as interpreted after the October 1958 testing point towards this also. The national norms have been analyzed in terms of the following eleven areas: (1) adjustment to school work, (2) personal-psychological relations, (3) social-psychological relations, (4) curriculum and teaching procedures, (5) courtship, sex, marriage, (6) social and recreational activities, (7) health and physical development (8) morals and religion, (9) home and family, (10) finances—living conditions and employment, and (11) future—vocational and educational. The results of the administering of the check list in October 1958 showed the eighth grade conformed to national norms already in all of the above areas with the exception of numbers 1—4—9—10. The responses to areas 1 and 4, (adjustment to school work, and curriculum and teaching procedures) seemed to "indicate a much greater concern over school work and progress than is generally shown. Areas 9 and 10 (home and family, and finances—living conditions and employment) seemed

TABLE II

FREQUENCY DISTRIBUTION OF SCORES ON THE ARITHMETIC ACHIEVEMENT TEST

Arithmetic Achievement	Total No. of Cases		Class Numbers													
			1 S		2 E		3 E		4 E		5 A		6 A		8 A	
	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3
Total Score	206	206	23	23	27	27	34	34	31	31	34	35	34	33	23	23
25	1	3				2	1									1
24	5	21			1	4	4	11		3		2		1		
23	7	16				4	5	5	2	5				2		
22	17	25			5	3	10	6	1	5		3		6	1	2
21	17	15			3	3	7	2	3	3	1	4	3	2		1
20	16	19	1	2	3	6	3	4	2	2	2	4	4		1	1
19	14	21		2	4	1		1	3	6	1	4	4	4	2	3 mean T3
18	12	16	1		1	1	1	1	2	4	4	4	1	3	2	3
17	20	12	1	5	2	1	1	1	7		3	1	3	2	3	2 mean T2
16	16	9	1		4	1			2	1	4	3	3	3	2	1
15	16	12	3	1							8	3	4	4	1	4
14	7	6	1	1					3		1	2	2	3		
13	13	4	2	2					1		1	2	5		4	
12	7	5	3	1					1		1	1	1	1	1	2
11	6	1	3								1	1	1		1	
10	5	3	1						2		1	1	1			2
9	4		1								1				2	
8	3	4	1	2							1		1	1		1
7	4	3	1	2							2			1	1	
6																
5	1	2	1	2												
4		1		1												
3	1		1													
2																
1																
0																
No Score	14	7	1	2	4	1	2	3	2	1	2	0	1	0	2	0

Key: S—Slow Average
A—Average
E—Enriched
T2—Results on Second Testing
T3—Results on Third Testing

TABLE III

FATHER'S OCCUPATION COMPARED WITH ARITHMETIC ACHIEVEMENT

Arithmetic Achievement	Total Students Responding		Number of Students Classified by Father's Occupation															
			Professional		Semi-Prof., Owner of Business		Admin., Manager		Sales, Clerical, Service		Skilled Worker		Semi-Skilled		Unskilled		Father Decesaed	
	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3
Totals Quartiles	192*	200*	27	29	51	58	20	19	25	22	33	33	19	20	15	17	2	3
Q1	48	50	8	13.2	17	14.6	5	4.2	6	4.8	7	8.8	4	3.6	1	.4	0	.4
Q2	48	50	8.75	7.8	10.50	17.4	3.25	4.8	7.25	3.2	11.75	9.2	3	5.4	2.50	2.6	1	.6
Q3	48	50	7.50	5	10	13	6.75	6	5.50	8	9.25	8	4.75	7	3.25	3	1	0
Q4	48	50	2.75	3	13.50	13	5	4	6.25	6	5	7	7.25	4	8.25	11	0	2

* Totals vary because all 208 students were not available for both testings due to school absence.
Key: T2—Results on Second Testing
T3—Results on Third Testing

to indicate that home and family conditions are very good compared to national trends."[5]

This concern over school work seems to be a result of pressure brought to bear on the child by his parents. In other words, the attitudes of the parent are influencing the performance of the child.[6] This study indicates that where there is a high degree of success on the part of the parent, one would expect to find a high degree of achievement in school on the part of the child.

There is a relationship between achieving and having a parent who was foreign born.

The fact that a high relationship exists between parents who were foreign born and arithmetic achievement is another indication of the same thing. The children of the foreign born who have made a successful adjustment to a new environment would, no doubt, be encouraged to do as well as their parents had. One would expect certain ideals, notably industry, to be held up by a person who had made a good adjustment to a new world.

It might be well to reiterate here that while these conclusions are stated with strong conviction, they are not conclusive enough to make broad generalizations and universal applications. However, they are valuable for two reasons: (1) they are reliable for the use in the community studied and for

[5] William Caldwell, *Results of Mooney Check List*, unpublished mimeographed report, October, 1958, p. 2.

[6] Attitudes do affect achievement in arithmetic; this was a result of a pilot study conducted at the University of California in 1955. This study used interviews with seventeen students for the source of its data. According to the results of this study, "The data indicated that parents determine the initial attitudes of their children and affect their achievements in arithmetic and mathematics. Three factors seemed to affect both attitudes and performance in these subjects: parental expectation of children's achievement, parental encouragement regarding these subjects and parent's own attitudes toward this area of the curricula. Assuming ability is present, the children of parents who expect them to do well in arithmetic and mathematics tend to do better than children whose parents expect them to do poorly in such subjects." Thomas Poffenberger and Donald A. Norton, "Factors Determining Attitudes Toward Arithmetic and Mathematics," THE ARITHMETIC TEACHER, April, 1956, pp. 113–116.

TABLE IV

BIRTHPLACE OF PARENTS COMPARED WITH ACHIEVEMENT IN ARITHMETIC

Arithmetic Achievement	Students Responding		No. of Students with: Father and Mother Born in N.Y.C. Area		One Parent In N.Y.C. and Other in U. S.		Both Mother and Father in U. S. but not N. Y.		One Parent Born in U. S. Other Foreign Born		Both Foreign Born		Not Reporting		One U. S. Other Not Reporting	
	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3
Totals Quartiles	192*	200*	123	130	15	14	5	8	22	21	4	5	14	14	9	8
Q1	48	50	32	36.2	1	2	2	1	5	5.6	1	1	3	2.2	4	2
Q2	48	50	28	29.8	3	0	2	2	10	8.4	0	1	3	3.8	2	5
Q3	48	50	37	31	2	5	0	3	1	4	1	2	5	4	2	1
Q4	48	50	26	33	9	7	1	2	6	3	2	1	3	4	1	0

* These totals vary because all 208 students were not available for testings.
Key: T2—Results on Second Testing
T3—Results on Third Testing

TABLE V

NEWSPAPERS READ BY FAMILY COMPARED WITH ARITHMETIC ACHIEVEMENT

Arithmetic Achievement	Total		Times-Tribune		Post Journal	Telegram Amer.	Press-Newsday		News-Mirror		Not Reporting	
	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3	T2	T3
Totals Quartiles	192*	200*	67	71	21	23	85	90	17	14	2	2
Q1	48	50	29	27.2	4	4.8	12	15.6	3	2.4	0	0
Q2	48	50	17.50	21.8	4.50	4.2	20.50	18.4	4	4.6	1.50	1
Q3	48	50	10.75	13	8.25	6	26.50	29	2	1	.50	1
Q4	48	50	9.75	9	4.25	8	26	27	8	6	0	0

* Totals vary because all 208 students were not available for both testings due to school absence.
Key: T2—Results on Second Testing
T3—Results on Third Testing

the group studied and (2) they provide a pilot study which when taken together with other such studies a broad theoretical framework may be developed.

EDITOR'S NOTE. If the parents read *The New York Times* the child is apt to do well in mathematics. Granted that this is true it is not a causal relationship of *The Times* and the mark in mathematics. Rather, it is probably evidence that factors that produce the one also produce the other. However, it is useful for a school to know the kinds of factors that tend to differentiate the pupils in their interest and ability in mathematics. How early do these factors appear as determinants in a child's life? Our work in the schools is with all levels. There will always be differences in native ability and in the nurture of this ability in the home and community. We should do with each child the best we can so that he may fulfill his optimum role in society.

"Plus" Work for All Pupils

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The recent concentration on special work in arithmetic for special children may turn out to be unhealthy. If the considerable effort of mathematicians and educators in formulating a major revision of the math curriculum is restricted to educating the talented child, much of value may be lost in the rush to protect our national honour. Children who are "ungifted" by present measuring devices (I.Q.s, standardized achievement tests, etc.) frequently reveal considerable "gifts" under stimulation, and in some cases without it. Many of us are hopping on the "gifted" bandwagon so enthusiastically that we run the risk of letting the new mathematics pass over the heads of our ordinary classes and our ordinary teachers. In an age where one must run as fast as one can in order to stay in the same place this would be indeed unfortunate.

A recent article in the Arithmetic Teacher (November 1959) by Eunice Lewis and Ernest C. Plath describes an experiment with a group of 11 fifth and sixth grade children who had been selected on the basis of recorded I.Q.s above 130 and grade placement scores of 6.5 or higher in arithmetic achievement tests. These children responded well to the challenge of questions requiring original mathematical thinking and the authors were pleased with the progress the children made during the semester. I thought that it would be interesting to try this material with a heterogeneous group of children I see one 45 minute period a week in order to compare their performance with the gifted group. The results were sufficiently interesting to warrant further experimentation, and eventually I tried the same material with heterogeneous groups of 7th and 8th graders as well as with a combined 5th and 6th group. In order to eliminate the teacher factor I used homogeneous "enrichment" groups (a combined 5th & 6th, and a 7th) as a control. The "enrichment" groups had been selected on the basis of I.Q.s above 130 and reading ability at least 2 years above grade level. In addition I tried out the same material with a class of 20 college girls, juniors and seniors, who were preparing for certification as elementary school teachers. If the results are at all indicative of wider applications, we should be most unwise to restrict the new curriculum to programs for "gifted" students.

In the heterogeneous groups with which I worked the I.Q. range, as measured by the Wechsler Intelligence Scale for Children went from about 90 to over 160. The measured grade level in arithmetic (Stanford Achievement Tests, Intermediate Battery) ranged from 4.3 to 9.0 for the 6th graders. I used the same problem with each group; "find the sum of the first 60 odd numbers." The only condition was that the solution had to be easy. The heterogeneous groups came up with answers in a very short time and each amateur mathematician was able to describe his reasoning with remarkable clarity. A pattern of response developed, which applied equally well to the homogeneous control groups, whereby the first breakthrough (no hints were given) was the discovery that the 60th odd number was 119. This was explained by the fact that there are as many even numbers as odd, therefore there are 120 all together, therefore the 60th odd number is 119. Having thus set the limits of the problem the solutions rolled in.

One child suggested that the "average" odd number would be 60. Objection from the class: 60 is an even number. Answer from the defender of this method: well an average doesn't have to be the same kind of number; for example, the average of 2 and 3 is $2\frac{1}{2}$. Class satisfied, but not the teacher, who asks: How do you know 60 is the average? How do you find the average (mean)? No guessing allowed; you must be able to prove your method will always work. But we record the solution anyway: The "average" is 60 and there are 60 odd numbers so their sum is 60 times 60 or 3600.

By this time other hands are raised. One of the faster children has noticed that:

$$\begin{aligned} 1+119&=120\\ 3+117&=120\\ 5+115&=120\\ &\cdots\cdots\\ 59+61&=120 \end{aligned}$$

and that these 30 pairs of odd numbers add up to 30 times 120 or 3600. Independent confirmation. First pupil: "See, my average was right!" I make a note to deal with this later. Light dawns all over the room. "This problem isn't so hard."

It is now time to give a hint about the method the teacher has had in mind all along. The class is asked to consider:

$$\begin{aligned} 1 \quad + \quad 3 &= \ 4=(2\times 2)^{*}\\ 1+3+5 &= \ 9=(3\times 3)\\ 1+3+5+7 &= 16=(4\times 4) \end{aligned}$$

"The first five odd numbers will add up to 5×5 or 25," says one child. Another sees that the first 60 odd numbers will add up to 60×60, "The same answer!" Another, because we have worked with unknowns in this group says, "Any number of odd numbers will add up to that number squared," and we come to the general case: $n\times n$ or n^2.

How do we know that the class has gotten the point? We give them another problem of similar type. What is the sum of the first 80 even numbers? Quick answers: 80×80; 6400. No. Some thought. Then a child who has never done well in arithmetic, who has been the bane of the arithmetic teacher's existence, raises a bright hand and bluts out: "The 80th even number is 160, and there are 40 pairs, and 2 and 160 is 162, and it's 40 times 162. It's 6480!" This child is quicker to grasp this abstract transfer than the brightest child in the class. How about the other method? What do small sums show?

$$\begin{aligned} 2+4 &= \ 6=2\times 3\\ 2+4+6 &=12=3\times 4\\ 2+4+6+8 &=20=4\times 5\\ 2+4+6\cdots+n &=n(n+1)\\ 2+4\cdots+80 &=80\times 81=6480. \end{aligned}$$

By this time even the dullest child is intrigued. This is not magic. This is something you can figure out. Even Johnny can do it. I'll try too. Another question: How would you find the sum of all the numbers from 1 to 100? Gasp. Quick answer: It's the same as the sum of the odd numbers added to the sum of the even numbers. It's 100×100 plus 100×101. No it isn't, the first hundred numbers only have 50 odd numbers and 50 even numbers. Yes, that's right, it's $50\times 50+40\times 51$; it's 5050! Students talking, not teacher. I suggest trying the other method. It doesn't work. No one sees the pattern:

$$\begin{aligned} 1+2 \quad &= \quad 3=\frac{2\times 3}{2}\\ 1+2+3 \quad &= \quad 6=\frac{3\times 4}{2}\\ 1+2+\cdots+n &=\frac{n(n+1)}{2} \end{aligned}$$

However there is a simple method developed in a manner comprehensible to 7th graders and used recently by W. W. Sawyer in the *Mathematics Student Journal* (Vol. 6, No. 1, et seq.) which could be used with some groundwork at this point. This method (finite differences) would be particularly enlightening to a pupil who had struggled

* The second step is discovered by the class.

(*Concluded on page 246*)

The Relationship Between Arithmetic Achievement and Vocabulary Knowledge of Elementary Mathematics*

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TEACHER TRAINING DEPARTMENTS in elementary mathematics often assume that if a prospective elementary teacher attains a high score on an achievement test in arithmetic, that person also possesses a high level of attainment in vocabulary knowledge of elementary mathematics. This assumption may have a bearing on teacher training curricula and course content. The purpose of this study was to determine objectively the degree of relationship between achievement in elementary arithmetic and vocabulary knowledge of elementary mathematics as possessed by prospective elementary teachers.

A total of 52 students enrolled in Mathematics 202 (Arithmetic for Teachers) at the University of Illinois (Urbana, Illinois) was used in the study. All of the students in the study were enrolled in the College of Education. Seventy-three per cent of the students were seniors. Six per cent of the students had had student teaching experience before taking the course and 2% of the students had had teaching experience in the field. All of the students in the study had had at least two years of high school mathematics. Thirty-six per cent had had more than two years of high school mathematics. Twelve per cent of the students had had college mathematics. The I.Q. of the students ranged from 94 to 143 with a mean of 118.

Three tests were given at the beginning of the second semester, 1958–9. The tests given were as follows:

1. Achievement Tests in Elementary Arithmetic (Achievement)
2. Vocabulary Test in Elementary Mathmatics (Vocabulary)
3. *California Short-Form Test of Mental Maturity*, S Form 1957 (Mental Maturity)

A forty item achievement test in elementary arithmetic was developed for the study. The items comprised operational skills in performing conventional computations in whole numbers, common fractions, decimal fractions, per cents, and verbal problems. Examples of the problems in the test are as follows:

1. $809\times457=$ ______
2. $12\times10\frac{3}{5}=$ ______
34. $15=12\%$ of ______

Logical validity was established by showing that the test corresponded to the definition of the traits intended to be measured. It was further established that the types of problems in the arithmetic achievement tests were given in four recently published elementary mathematics series (Grades IV–VIII). In order to establish reliability, a variation of the split-half method, termed the parallel split was used to estimate the coefficient of equivalence. The test papers from the pilot study (I Semester, 1958–9) were analyzed to determine how many persons had each item correct. The items were then divided into two equal groups such that the items in the two halves were matched in difficulty and in content. The group of papers in the study were scored on the two half-tests and

* A condensation of certain sections of a doctoral thesis, University of Illinois, 1959.

Guttman's formula was applied. Using $N=52$, the coefficient of reliability of the achievement test in arithmetic was .92.

A vocabulary test was also developed for this study. It was a recall type of test consisting of fifteen selected terms from elementary mathematics. The fifteen terms were selected from the list of mathematical terms introduced in one of the leading series, *Making Sure of Arithmetic*. The mathematical terms introduced in this series were placed into thirteen categories. The following seven categories were used in this study:

1. Terms referring to the four fundamental processes of operation
2. Terms referring to the structure of the number system
3. Terms referring to computational steps
4. Terms referring to quantity, order, or comparison
5. Terms referring to algorisms, terms of a fraction, or operational directions (other than the four fundamental processes of operation)
6. Terms referring to figures
7. Terms referring to measurement

The split-half technique was used to determine the reliability of the vocabulary test. The product-moment correlation coefficient corrected by the Spearman-Brown formula was .71.

The purpose of the vocabulary test in elementary mathematics was to evaluate the student's ability to explain in detail the fifteen terms. Wherever applicable, this explanation was to include (in addition to a definition) different meanings, uses, relationships, generalizations, principles, symbols, equivalents, explanations, and algorisms. Even though the vocabulary test was to be a recall type of test, the testees had to have some idea as to the scope and explanation desired. After revisions, it was found that the following directions fulfilled the criteria set up:

> The following is a list of terms in elementary school mathematics (grades 1–8). Explain these terms as they are used in the elementary grades in as much detail as you can. Where applicable, your explanation should include different uses, relationships, mathematical symbols, etc. Use the language level of a child of normal intelligence in the elementary grades. You have the entire period (100 minutes) for this test.

The fifteen selected terms were as follows: subtraction, one hundred, division, square, tens' place, ratio, minuend, fraction, multiplication, quart, per cent, cubic unit, carry, decimal fraction, and dividend. A score sheet was set up giving the important knowledge associated with each term. Points were allocated for the particular aspects of knowledge. For example, 13 points were distributed for the knowledge dealing with the definition, uses, symbol, relationships, principles, and generalizations associated with the term "subtraction." The total number of points on the score sheet was 101. The score sheet was validated by having four judges grade a random sample of ten papers. A coefficient of concordance was computed and converted to a χ^2 value. The null hypothesis was rejected at the 1% level of significance which indicated with considerable assurance that the agreement among the five graders was higher than it would have been by chance.

In addition to the major problem of the study (to determine the degree of relationship between arithmetic achievement and vocabulary knowledge of elementary mathematics), there were related problems. These will also be considered in the summary and conclusions. The conclusions are given realizing the limitations of generalizing on a small sample of 52 students.

Summary and Conclusions

1. The relationship between arithmetic achievement and vocabulary knowledge of elementary mathematics was found to be .53. This product moment correlation coefficient is significant at the 1% level. The coefficient of determination, r^2, is .28. Thus 28% of the variance in one is associated with the variance of the other.

2. Using vocabulary scores as a function of achievement scores, the standard error of estimate was 5.1. Using achievement scores as a function of vocabulary score, the stand-

ard error of estimate was 7.1. Thus there is a greater error of estimate in predicting achievement scores from vocabulary scores than in predicting vocabulary scores from achievement scores.

3. A student attaining a high score on an achievement test may not necessarily attain a high score on the vocabulary test. Since it is important to ascertain the level of achievement in both areas, a testing program for prospective elementary teachers should include more than just an achievement test in elementary arithmetic.

4. Using vocabulary scores as a function of achievement scores, the correlation ratio was found to be .64. The hypothesis $\eta_{yx}=\rho$ was accepted (5% level of significance). This indicates that there is a linear relationship between arithmetic achievement and vocabulary knowledge of elementary mathematics.

5. The correlation between arithmetic achievement and mental maturity was found to be .42. This correlation coefficient is significant at the 1% level but it doesn't indicate an extremely high relationship.

6. The correlation between vocabulary knowledge of elementary mathematics and mental maturity was found to be .25. This correlation coefficient is not significant at the 5% level.

7. The findings showed that there was a significant relationship between achievement and vocabulary knowledge and between achievement and mental maturity. There was no significant relationship between vocabulary and mental maturity. This indicates the possibility that achievement and vocabulary have a factor (or factors) in common, achievement and mental maturity have a factor (or factors) in common, and vocabulary and mental maturity have no factor in common.

8. The correlation between arithmetic achievement and vocabulary knowledge in elementary mathematics, holding mental maturity constant, was found to be .49. This partial correlation coefficient is approximately the same as that of the product-moment correlation coefficient of .53.

9. The correlation between arithmetic achievement and vocabulary knowledge was found to be .39 for the students above the median on the mental maturity test (median 95.7, raw score). This correlation is barely significant at the 5% level. For the students below the median on the mental maturity test, the relationship was found to be .61, significant at the 1% level. The difference between the two correlation coefficients is not significant at the 5% level. This indicates that it is inappropriate to assume that the relationship between arithmetic achievement and vocabulary knowledge is significantly higher for students below the median on the mental maturity test than for those above the median on the mental maturity test.

10. The range for the arithmetic achievement test was 18%–100% and the range for the vocabulary test in elementary mathematics was 25%–50%. The standard deviations for the two tests were 8.3 and 5.9 (computed on raw scores) respectively. Thirty students out of 52 students received a score below 75% on the arithmetic achievement test. Assuming that prospective elementary teachers should have a high level of operational skill in elementary arithmetic, the above suggests the necessity for the recognition of individual differences in elementary teacher training courses.

EDITOR'S NOTE. The vocabulary of elementary school mathematics is normally more associated with the ideas and concepts of mathematics than with computations. One could learn a great deal of computation without much use of language. That however, is not desirable. Certain processes and symbolic representations such as common fractions have names for the elements as well as the whole and these should be learned in order to discuss and describe. One might expect that the correlation between achievement and vocabulary would be higher than the .53 found by the author. This suggests that we ought to give attention to the development of ideas and the words that convey them. Assuming that the students in the sample used are a fair sample, it is a good thing for many of them to have a class in arithmetic when one who is a senior has a score as low as 18%. It is these people who know little mathematics and who have a fear of the subject that are most apt to be harmful to a good program.

In-Service Research in Arithmetic Teaching Aids

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The amount of concern teachers have for the proper selection of teaching aids ranges from those who accept "gadgetry" without question to those who give careful consideration to their selection and use. This is a report of the manner in which one group of teachers undertook an evaluation of arithmetic teaching aids.

A representative of the Monroe Calculator Machine Company introduced the *Educator*, a hand-operated calculator, to a workshop group of fifth grade teachers in Midland, Texas. After his visit teachers raised a number of questions relative to its use in classroom instruction. It was decided that a study should be undertaken to determine the effectiveness of this aid in comparison to others which might be used. This is a report of that study.

The Problem

The problem of the study was to determine the relative effectiveness of three different types of aids in improving the mathematical understanding and achievement of fifth grade pupils and their teachers. The following hypotheses were established: (1) The use of the Educator, the Abacounter (an abacus type aid available through Creative Playthings, Inc.), and teacher-made aids (the place-value chart and the number line) will make a difference in pupils' mathematical understanding and achievement. (2) The use of different mathematical teaching aids will make a difference in teachers' mathematical understanding.

Method

Seven school principals and three consultants assisted teachers in designing the study which included twenty-four fifth grade classes. Three groups of six classes each used the teaching aids. Each group used one of the three types. A fourth group of six teachers served as a control group only for data gathered on teachers' mathematical understanding. Twenty-four teachers volunteered to participate in the study and through a system of random selection, each of these teachers was assigned to one of these four groups.

Each of the six classes using the Educator (Calculator) was supplied with three of these machines. Each of the six Abacounter classes were supplied with a large teacher-demonstration Abacounter and ten small pupil desk-size Abacounters. Each of the classes using the teacher-made aids used both the place-value chart and the number line. A local resource leader, either a principal or an elementary supervisor, was named to each of the three groups and regular meetings were held at which time instruction in the use of the aid was given and teachers shared with others the techniques they had found to be effective with their groups.

The experiment continued over a five-month period and pupils in the eighteen experimental classes were given different forms of the *California Arithmetic Test* as pre- and post-tests from which mean increase scores in arithmetic reasoning, arithmetic fundamentals, and total arithmetic achievement were determined.

To gather data on the second hypothesis a test of mathematics understanding originally developed by Bliesmer, at the University of Virginia, and revised and edited by the writers, and a colleague at the University of Texas, was administered to the twenty-four teachers before the project got underway and again in another form five months later at the end of the project.

Results

The analysis of variance technique was used in analyzing pupil data. Independent variables included method and level of arithmetic achievement. The method variable included the three teaching aids. Pupils were divided into upper, middle, and lower achievement levels in terms of initial scores on the *California Arithmetic Test* to determine the extent to which the effectiveness of specific aids varied with differing achievement level groups. Dependent variables included increase between pre- and post-test scores in arithmetic reasoning, arithmetic fundamentals, and in total arithmetic achievement. These analyses are presented in Tables I, II, and III.

TABLE I

MEAN INCREASE IN ARITHMETIC REASONING SCORES OF 270 FIFTH GRADE PUPILS

Method	Arithmetic Achievement			Total
	Upper	Middle	Lower	
EDUCATOR	13.03	13.90	12.27	13.07
Abacounter	13.67	12.73	13.17	13.19
Teacher-made	9.33	11.93	11.83	11.03
Total	12.01	12.86	12.42	12.43

Analysis of Variance

Source of Variation	d.f.	Sum of Squares	Mean Squares	F	P
Arithmetic Achievement (A)	2	32.1	16.05		
Method (M)	2	283.9	131.50	6.38	.01
AM	4	447.3	111.83	5.42	.01
Deviation Among Persons	261	5382.9	20.62		
Total Variation	269	6126.2			

TABLE II

MEAN INCREASE IN ARITHMETIC FUNDAMENTALS SCORES OF 270 FIFTH GRADE PUPILS

Method	Arithmetic Achievement			Total
	Upper	Middle	Lower	
EDUCATOR	10.07	9.40	8.23	9.23
Abacounter	12.30	12.50	10.13	11.64
Teacher-made	10.33	8.20	8.57	9.03
Total	10.90	10.03	8.98	9.97

Analysis of Variance

Source of Variation	d.f.	Sum of Squares	Mean Squares	F	P
Arithmetic Achievement (A)	2	166.8	83.40		
Method (M)	2	380.2	190.10	3.04	.05
AM	4	613.3	153.33	2.44	.05
Deviation Among Persons	261	16,407.5	62.86		
Total Variation	269	1,160.3			

Among achievement level groups there were no significant differences in mean increase in any of the three dependent variables including arithmetic reasoning, arithmetic fundamentals, and total arithmetic achievement. However, relative to the "methods variable," significant differences resulted when each of the dependent variables was analyzed. Mean increases in *arithmetic reasoning* of pupils who had used the Educator or the Abacounter were significantly greater at the 5 per cent level than for those using the teacher-made aids. Mean increases in *arithmetic fundamentals* were significantly higher at the 5 per cent level for those pupils using the Abacounter than for pupils using other aids. Mean increases in *total arithmetic* achievement for pupils using either the Educator or the Abacounter were significantly greater at the 5 per cent level than for those using the teacher-made aids. Significant interactions between independent variables indicate that particularly for *arithmetic reasoning* the teacher-made aids were of most value to lower-achievement-level pupils.

The results of the tests of understanding taken by the teachers of these three groups of pupils and the group of six control teachers revealed that all groups made gains in their understanding of mathematical concepts. However, only the gains made by the teachers using the Educator and the Abacounter were significant at the 5 per cent level. These results are reported in Table IV.

TABLE III

MEAN INCREASE IN TOTAL ARITHMETIC ACHIEVEMENT SCORES OF 270 FIFTH GRADE PUPILS

Method	Arithmetic Achievement			Total
	Upper	Middle	Lower	
EDUCATOR	23.13	23.37	20.13	22.21
Abacounter	25.63	25.23	22.97	24.61
Teacher-made	19.67	20.13	20.07	19.96
Total	22.81	22.91	21.06	22.26

Analysis of Variance

Source of Variation	d.f.	Sum of Squares	Mean Squares	F	P
Arithmetic Achievement (A)	2	196.1	98.05		
Method (M)	2	975.7	487.85	4.47	.05
AM	4	1,298.7	324.68	2.97	.05
Deviation Among Persons	261	28,479.4	109.12		
Total Variation	269	30,949.9			

TABLE IV

T-TEST OF SIGNIFICANCE OF MEAN GAIN IN ARITHMETIC UNDERSTANDING OF FOUR GROUPS OF FIFTH GRADE TEACHERS

Method	Number	Mean	Standard Deviation	Standard Error Mean	T	P
Educator	6	9.17	6.09	2.73	3.36	.05
Abacounter	6	7.33	3.15	1.86	3.95	.02
Teacher-made	6	2.17	4.88	2.18	.99	—
Control	6	4.50	6.80	3.04	1.48	—

Conclusions

Pupil groups which used the Educator and/or the Abacounter made greater gains in arithmetic achievement than those pupils using teacher-made aids. Teachers of these groups likewise increased in their mathematical understanding significantly more than others. The specific teaching aids used in these classrooms did make a difference in the achievement of pupils. For the most part, specific aids which were successful with pupils of one achievement level were effective with pupils of other levels. Perhaps most important is the conclusion that something happened to the understanding and achievement of both teachers and pupils as this group of in-service teachers investigated the effectiveness of arithmetic teaching aids.

A number of questions may be raised. Is this pupil and teacher growth the result of a research approach and a research attitude on the part of teachers? Is this growth in teacher understanding the result of work with specific teaching aids or is it a result of teachers' involvement in research activity? Is the increase in pupils' reasoning and fundamental skills, which is apparently related to the growth made by their teachers, a function of the aid used or a function of the increased understanding of their teachers? How permanent are the changes made by teachers? Will future classes profit from the understandings developed by teachers in this project?

EDITOR'S NOTE. The authors ask whether the larger gains in mean growth might be due to the research attitude of the teachers. Such is often the case when a novelty element is introduced and the teachers themselves must do extra work to understand and carry out the use of the new procedure and instrument. It is probably fair to assume that the additional learning gained by these teachers will have values for other classes even though the experimental methods are not used. An experimental and research attitude is good for all teachers. When one ceases to question and to explore for new ideas, growth stops and retrogression probably begins. With "T" scores above 3 for both procedures using the Educator and the Abacounter no reasonable doubt for the values of these procedures should exist in terms of the experimental situation. There may be other materials and procedures that would yield an equal significance when the same amount of time and energy is expended.

"Plus" Work for All Pupils

(Concluded from page 239)

hard to detect the law by trial and error. A further advantage of following through with the method of finite differences would be that the teacher could effectively answer the question an 8th grader asked me: "How do you know that this pattern keeps going on?" Of course, we don't know that it does until we prove it, and the kind of questions we ask children in arithmetic should have answers we ourselves know how to prove.

The homogeneous control groups responded about as well as the heterogeneous groups, *but no better*. This is significant. What is equally significant was that the college girls responded about as well as the 5th and 6th graders but not as well uniformly. Some were too old to exhibit the enthusiasm that came naturally to the 11-year-olds. A sequal to the experiment (which continues, using the syllabus being prepared by Professor Davis of the Syracuse, N. Y. "Madison Project," *q.v.*) was the discovery of an 8th grader, the next week, that

$$1^3+2^3=(1+2)^2 \text{ etc.}$$

She had been given an assignment by her regular teacher to find the sums of cubes and this discovery gave her a very easy method for completing the assignment. The 8th grade now wants a proof for this. I can't see how to do it without using summation notation. Can you?

EDITOR'S NOTE. It is heartening to have people all over the country experimenting with different materials and with levels other than those tagged as "gifted." The editor has long cherished an old proverb: "What one fool can do, another may learn." We so often shy away from ideas because they are new or strange to us. Mr. Clarkson noted that his college students were much like fifth and sixth graders but lacking in spontaneity. Having found that pupils of a certain age level can discover and learn some interesting topic in mathematics, we are still faced with the decision as to whether this topic is more fruitful than some other which it may replace. The decision is not easy. Perhaps, at least for some pupils, the discovery and formulation of a principle such as $S=n/2(a+l)$ is much more worthwhile than the mastery of $7\times 9=63$. But we must not be hasty in recasting the curriculum for the elementary schools nor should we refuse so to do when evidence clearly indicates revision.

Filling a Gap in Subtraction

Olivia H. Baldwin, *Supervisor*
Negro Schools, Palm Beach County, Fla.

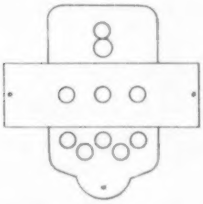
Several gaps appear in the teaching of concepts at the primary level when there should be continuity and sequential development step by step. These gaps must be filled in, rather than pulled together if concepts are to be meaningfully clear and processes understood and functional. Observations and outcomes in pupil performance point up the fact that our teachings of the substraction concepts lack wholeness.

A device—Basic Subtracto-Addo Fact Boards—is proposed as an aid. The boards give a visualized, realistic approach that will aid in the discovery and understanding of the concepts and processes involved in subtraction. At the same time these boards show the positive relationship of subtraction to addition. They lessen the confusion caused by the transition from the semi-concrete experiences to abstract thinking.

Because the "take-away" concept of subtraction lacks wholeness as now taught by many, both the concept and the process become confused. Children are taught that subtraction means "take away." This "taking away" means separating or regrouping, but the visualized process proves a barrier to understanding because the quantity of blocks or circles though crossed out or blocked out remain visible—an attempt to teach that a thing is and is not at the same time. Barriers or blocks to understanding and meaning thus arise.

Incompleteness in concept also arises when children are taught to think of subtraction only as a process of "take-away." This incompleteness together with the haziness of the "take-away-though-I-still-see-it" concepts serve to retard sequential and continuous meaningful number development. Abstractions, techniques and symbolisms are not as easily understood as thought or, perhaps, desired. To "take away" means to make a part or all of a whole thing disappear. Johnnie had two peaches; he ate one or Betty have five pennies; she lost two. Both a peach and two pennies are gone. They have disappeared. The difference that remains might easily be the result of counting, estimating, computing; of comparison, addition or take-away. Three subtractive concepts—*comparison*, *addition*, and *take-away* involved in the subtractive idea need be visualized for meaning both of the concept and the process. These must be taught if understandings and skills are to be meaningfully developed. These must also be interrelated and understood by the children and must relate to the children's concerns in many and varied social situations.

The following drawings are two of the series of the subtraction concept boards and are designed to relate the subtractive process to addition process.

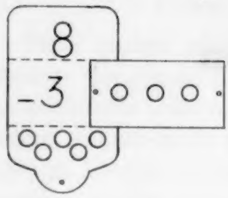


The top of the board shows the total from which the number is to be taken

Removable slat which shows the number to be taken away

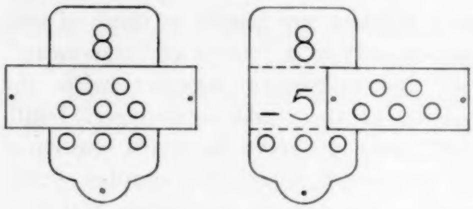
What is left when the removable slat is taken away.

As the board appears now, the symbol 8 is given with the meaning in circles. The abstract symbol 8 is the name of the number of circles with which we work. (semi-concrete). That eight things are needed to make eight is the desired understanding.



B. Underneath the removable slat is the symbol and sign of what is taken. This meaning of the sign for "take-away" must also be taught.

The board now becomes more abstract and appears as illustration (B). The symbol -3 takes the place of the 3 circles, thus associating the symbol and the conceptual meaning of 3. With the slat taken away, we have actually removed (taken away) 3 circles from 8 circles, leaving 5 circles.



Or we may have taken 5 circles from 8 circles leaving 3

Should the meaning of 3 or 5 not seem clear the slat might be again slipped partially in the slot beside, but not covering the 3 or 5 while the meaning between the symbol and the circles again is explained or clarified.

8 circles, take away 3 circles, leaves
o o
o o o (5) circles; and 8, take away 5 circles, leaves o o o (3) circles.

8	8
−5	−3
o o o	o o o o o

At this point be sure to make clear the visual concept of "take-away." Children should not be forced to memorize the meaningless facts and rules of any process until repeated experiences have readied them for meaning and use of the concept.

We need to show to children and have them understand that we can also find the difference—how many is left or gone—as we subtract. We began to compare subtractively. Though much used, the *difference* concept of subtraction does not involve the thinking as that of "take away." The concepts of larger, smaller, how much more, or how much less or how much must be added enter and must be made meaningful in many and varied social experiences. Such comparisons make the meaning of "difference" clear.

1. Replacing slat to give the picture of 8 being composed of 8 ones.
3. How many circles do we have? 8
4. The number at the top of our board is named 8
5. How many circles must we have to make 8
6. 8? Yes, each *one* of the 8 circles is needed to make 8.
7. How many ones then are needed to make 8?
8. (Pulling slat)—How many are we taking away?

8 take away 3 = 5

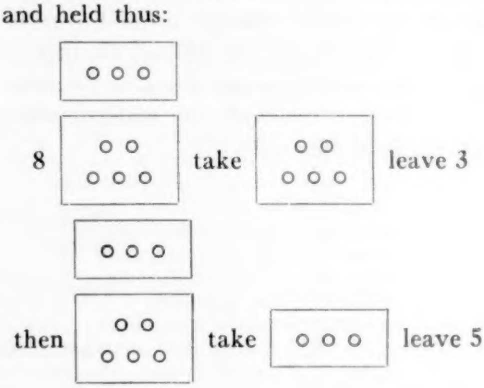
$$\begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array}$$

The parallel board is

$$\begin{array}{r} 8 \\ -5 \\ \hline \end{array}$$

and the process for teaching would be the same as above.

With the slats removed from both boards and held thus:



New concepts at this point are possible through questions, answers and manipulation of slats.

D. With slats in reversed positions the *difference* concept might be visualized and taught. The following suggested questions or statements will illustrate:

1. We have five (5) circles, how many more are needed to make 8? ____
2. 8 is how many more than 3? ____
3. We have three (3) circles, how many more are needed to make 8? ____
4. 3 is how many less than 8? ____
5. 5 is how many less than 8? ____
6. How much must be added to 3 to equal 8? ____
7. How much must be added to 5 to equal 8? ____
8. 8 is how many more than 5? ____

E. Other Experiences:

8 children were at the chalk board. 3 girls took their seats. How many children were left at the board?

We have 5 fingers on one hand. How many more fingers are needed to make 8?

There are 8 of us at the table. There are 5 chairs. How many more chairs are needed? Can you show this with circles?

$$\begin{array}{r}8\\-3\\\hline\end{array}\qquad\begin{array}{r}8\\-5\\\hline\end{array}\qquad 8-3=\qquad 8-5=$$

F. On the back of the board are the accompanying addition facts necessary for complete, meaningful and social understanding. Manipulating the slats to show

$$\begin{array}{r}5\\+3\\\hline\end{array}\ \text{or}\ \begin{array}{r}3\\+5\\\hline\end{array}\ \text{or}\ 5+3=\ \text{or}\ 3+5=$$

we question: What added to 3 equals 8? What added to 5 equals 8?

$$\begin{array}{r}8\\-5\\\hline\end{array}\quad\begin{array}{r}8\\-3\\\hline\end{array}\qquad\begin{array}{r}3\\+5\\\hline\end{array}\quad\begin{array}{r}5\\+3\\\hline\end{array}$$

Thus we have the completed number story and a suggested device to illustrate a conceptual approach for better understanding.

The subtracto-addo fact boards are for use in grades 1–3 covering the facts to 10 and are followed through in grades 4–6 by reference boards of the ninety addition and subtraction facts so arranged that a teacher might use them with individuals, the class or small groups or with immature, average and advanced groups. They might be self teaching. The more difficult of the 45 basic addition and subtraction combinations are wheeled. This serves as a testing device. Accompanying these devices teachers are requested to create many and varied experiences that move from the simple to the complex as concepts are cleared and processes understood.

EDITOR'S NOTE. Certainly many pupils in the lower grades need the visualization of the subtraction process and some of these profit from doing the manual process of removing items. Mrs. Baldwin's "boards" are made of scraps of plywood and masonite which are found in the woodshop. It might be suggested that the numerical symbols be available on the lower section so that the complete direct comparison with circles or objects and the written symbol be evident. For perhaps twenty years the editor has used a framed piece of sheet metal painted with "blackboard paint" and wooden counters which contain small alnico magnets to illustrate not only numbers but addition, subtraction, multiplication, division, and fractional relationships. Since the metal is painted, the written record of what is being done may be made in chalk and thus a complete visual-manual demonstration and the symbolic record are immediately available. This item was described in THE ARITHMETIC TEACHER, Volume I, No. 1, February, 1954. As Mrs. Baldwin says, certain pupils need to be shown much more than others and phases of this work may need to be carried into the intermediate grades. Learning with understanding is our goal.

Newly Elected Officers

The following officers and directors assumed the responsibilities of their respective offices at the annual meeting of the Council at Buffalo in April.

President: Phillip S. Jones
Vice President for Senior High School: William H. Glenn
Vice President for Elementary School: Clarence Ethel Hardgrove
Directors: J. Houston Banks
Irvin H. Brune
Robert E. K. Rourke

"Interest with Interest"

MINNIE SCHLICHTING

University High School, Lincoln, Neb.

THE COMPUTATION OF interest is a perennial instructional "headache" for the arithmetic teacher. There are several different methods of finding the amount of interest, but the 60-day method is the one which is used most frequently. Advocates claim that this method is short, and indeed it is, so long as the rate is 6 per cent and the interest is to cover 60 days. But try to convince students that the 60-day method is shorter when the rate is 5 per cent and the time is 96 days!

The Basic Method

Invariably boys and girls groan at the 60-day method and beg to be allowed to use the original or basic method in which the principal is multiplied by the rate for one year. This amount is then multiplied by the number of days and divided by 360. Most students learned this method in their first introduction to interest. They are reluctant to try a new way unless they are convinced that the "shorter" method is actually shorter.

360-day Method

There is really a short method of computing interest. It is the 360-day method which has not been included in many textbooks but is used by banks. In this method, interest is computed by multiplying the principal by the number of days, dividing by 100 or placing the decimal point two places to the left, and then dividing as follows:

by 120	if interest rate is	3%
by 90	if interest rate is	4%
by 72	if interest rate is	5%
by 60	if interest rate is	6%
by 51.43	if interest rate is	7%
by 45	if interest rate is	8%
by 40	if interest rate is	9%
by 36	if interest rate is	10%

Thus, in computing the interest on \$258.50 for 5 per cent for 96 days, the following mathematical operations are necessary, when the 360-day method is used:

Multiply \$258.50 by 96	\$24,816.00
Divide by 100	248.16
Divide by 72	3.446 or \$3.45

Contrast this with the mathematical operations necessary in the 60-day method as follows:

Interest for 60 days at 6%	\$2.5850
Interest for 30 days at 6%	1.2925
Interest for 6 days at 6%	.2585
Interest for 96 days at 6%	\$4.1360
Interest for 96 days at 1%	.6893
Interest for 96 days at 5%	\$3.4467 or \$3.45

It is well for students to know the "why" of the 360-day method. This may be done by reviewing the "cancellation" method which they may have learned previously.

1. Set up calculation as follows:

$$\$258.50\times\frac{5}{100}\ (\text{rate})\times\frac{96}{360}\ (\text{days})$$

2. By cancellation, the following results are obtained:

$$\$258.50\times\frac{\cancel{5}}{100}\times\frac{96}{\underset{72}{\cancel{360}}}$$

This serves as a good way to introduce the 360-day method:

$$\frac{\$258.50}{100}\times\frac{5}{360}\left(\text{or }\frac{1}{72}\right)\times 96$$

Thus students will only need to learn to multiply the principal by the number of days, move the decimal point two places to the left, and then divide by 120, 90, 72, 60, 51.43, 45, 40, or 36 depending on the rate

of interest. These divisors need not be memorized but students can merely remember that the divisor may be found by dividing 360 by the rate of interest.

When the figures contain fractions, the 360-day method is more accurate than the 60-day method and requires just two mathematical operations instead of four. In classes where adding machines are used, students can be taught to do the multiplication by machine. The writer has found that when students are given a choice of method they always choose the 360-day method.

It is unfortunate that 360 is not easily divisible by 7 but the 360-day method may still be used by means of two additional mathematical operations as follows:

Multiply \$258.50 by 96	\$24,816.00
Divide by 100	248.16
Divide by 60	4.14
Add $\frac{1}{6}$ of \$4.14	.69
Interest at 7%	\$ 4.83

If students learn the 360-day method, they will truly study "interest with interest."

Editor's Note. Special methods of computing interest such as the "6% 60-day method" and the "360-day method" are usually employed in courses in business arithmetic where it is assumed the student should learn a method which he may have occasion to use repeatedly. The basic principle of computing interest which is expressed in the formula $I=PRT$ is very important in our society and should be understood and learned by all. Special methods may be learned by pupils under circumstances which dictate the need for a special method. For example, people who hold investments frequently use a method for determining approximate rate of yield by relating the annual dividend to 100. e.g. a yield of \$1.80 on a cost of 34 is approximately 5.4% because 100 is nearly 3×34. Similarly \$1 on 16 is a little over 6% because there are more than 6×16 in 100. But these special methods may be learned as occasion demands and they can be understood by one who understands the basic principle of interest. To many people today the calculation of rate is probably more important than the calculation of interest. It should also be recalled that much calculation is done by machine and many financial institutions have developed tables. For some pupils the use of tables, machines, and special methods provides an extension of the basic learning.

A Kit for Arithmetic

Julia Adkins
Central Michigan University, Mount Pleasant, Mich.

While teaching a course (Arithmetic Principles and Teaching) for prospective elementary teachers, I wished to acquaint the students with a philosophy of teaching for which the "discovery method" is an expression of the philosophy. I decided that an effective way to achieve this goal would be to use role playing. In doing this I would use with the students techniques that I hoped they would use later with their own students.

This plan presented a problem. If students are to discover relationships and facts for themselves, *each* student must have materials with which to work.

After thinking about the problem for some time, I decided that a kit for each student would be a solution. For my purpose the kits must be compact, uniform in size (for easy storage), and inexpensive.

The problem of a container for the materials was solved when I walked into a bookstore at the beginning of the Christmas season. The clerks had just finished arranging a display of Christmas cards, and the empty boxes were stacked on the floor. These boxes met my requirements—compact, uniform in size, and certainly inexpensive! The clerks were happy to have me relieve them of thirty-five of the boxes.

My next job was to decide upon the types of manipulative mathematical aids to place in the boxes. I started with the kinds of aids that one might use with children in the first grade (see accompanying picture). The following materials were used:

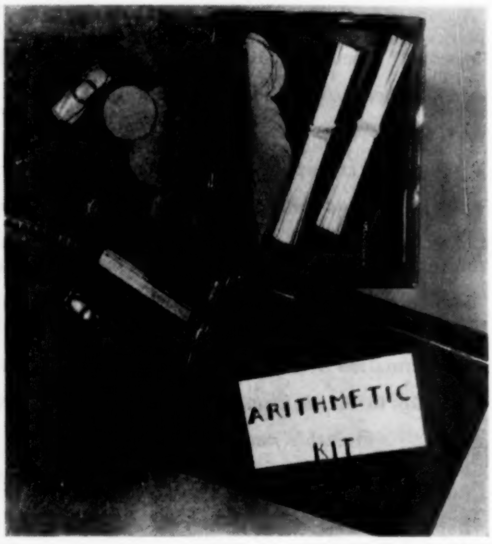
1. Paste sticks wrapped in bundles of ten each.
2. Cardboard circles to use as counters (these may be obtained from a school supply store).
3. Plastic toy automobiles.

As we progressed to other units and advanced in grade levels in our study of teaching techniques, I changed the materials in the kits.

When using the kits I did not wish to take class time to hand out and to collect them. Therefore, the kits were kept in a large cardboard box and on the days we needed them, this box was placed at the door. As each student entered the classroom, he picked up one of the kits; as he left the classroom at the end of the period, he returned the kit to the large box.

Now that each student had his own materials with which to work, my next task was to develop demonstration-size materials which closely resèmbled the students' materials. While they used toy automobiles, I had pictures of automobiles to place on the flannel board; while they used cardboard discs, I used magnediscs on sheet metal; and while they used bundles of paste sticks, I used bundles of large, red sticks.

With these types of materials—a kit for each student and demonstration material for myself—I felt that I was better equipped to convey to prospective teachers the principles of a philosophy of teaching on which the "discovery method" is based.



Research on Arithmetic Instruction—1959

J. Fred Weaver
Boston University School of Education

This summary of research on arithmetic instruction represents a continuation of a series begun three years ago in The Arithmetic Teacher,[1] and brings this series up-to-date through the calendar year 1959.

The present summary is organized in two major sections. The first of these is devoted to *published* research reports; the second, to *doctoral* theses and dissertations which have been reported in *Dissertation Abstracts* (as distinct from other published research reports).

Section I: Published Research Reports

The annotated references in this section include accounts of research on arithmetic instruction that were reported in periodicals, monographs, and the like during the calendar year of 1959. A serious attempt was made to make the listing as complete as possible. Omissions, if they exist, are unintentional.

In preparing the 1959 summary the writer applied the same criteria of delimitation that were used in previous years. Thus, published references included in the biobliography are restricted to: (1) normative and experimental studies which report specific data or findings on a problem associated with, or closely related to, mathematics instruction in the elementary school (grades K-6); and (2) bibliographies, summaries, and more or less critical discussions which relate in whole or significant part to such normative and experimental studies. It is hoped that these criteria were applied with reasonably acceptable fidelity when deciding whether or not to include a particular reference among those listed in this 1959 summary.

[1] Weaver, J. Fred. "Six Years of Research on Arithmetic Instruction: 1951–1956." The Arithmetic Teacher 4: 89–99; April 1957.

Weaver, J. Fred. "Research on Arithmetic Instruction—1957." The Arithmetic Teacher 5: 109–118; April 1958.

Weaver, J. Fred. "Research on Arithmetic Instruction—1958." The Arithmetic Teacher 6: 121–132; April 1959.

Annotated Listing of Published Research Reports

1. Bean, John E. "Arithmetical Understandings of Elementary-School Teachers." *The Elementary School Journal* 59: 447–450; May 1959.

Used Glennon's "Test of Basic Mathematical Understandings" as the criterion measure applied to 450 Utah elementary-school teachers. Found the mean rights-score to be about 66% of the total items in the test. Found a small cumulative increase in teachers' arithmetic understanding with greater teaching experience. Found that the amount of college preparation of a teacher was related directly to her level of arithmetic understanding: "Teachers who had done graduate work, for instance, averaged more than seven point above teachers who did not have a Bachelor's degree." Found intermediate-grade teachers scoring higher, on the average, than primary-grade teachers in regard to understandings of common and decimal fractions and the rationale of computation. Using a questionnaire "... devised to determine the relationship between test scores and various criteria deemed relevant to the problem, also found a positive relationship between teacher self-perception of arithmetic understanding, both before and after testing, and actual test scores; self-perception was somewhat more realistic after testing than before, however.

2. Bernstein, Allen. "Library Research—A Study in Remedial Arithmetic." *School Science and Mathematics* 59:185–195; March 1959.

Summarizes research on "remedial arithmetic," based on a bibliography of 32 references selected from over 200 relevant articles and research studies examined and evaluated by the author. The research is presented and discussed

in relation to three major categories: A. Remedial Teaching Projects; B. Error Diagnosis Studies; and C. Studies in Learning Theory. A final section of the report summarizes findings and conclusions that emerge from the research analyzed. Persons interested in the problem of "remedial arithmetic" undoubtedly will wish to read Bernstein's report in its entirety. (See the summary of "Six Years of Research on Arithmetic Instruction: 1951–1956" in the April 1957 issue of THE ARITHMETIC TEACHER for reference to Bernstein's own earlier research which was reported in the January 1956 and June 1956 issues of *School Science and Mathematics*.)

3. BOGUT, THOMAS L. "A Comparison of Achievement in Arithmetic in England, California, and St. Paul." THE ARITHMETIC TEACHER 6: 87–94; March 1959.

Reports a study based on administering to 524 children in St. Paul the same instrument used by Buswell in his earlier research. (See G. T. Buswell, "Comparison of Achievement in Arithmetic in England and Central California," *The Arithmetic Teacher* 5: 1–9; February 1958.) Compares findings from the two studies, and makes the following statement: "The writer feels that only one valid conclusion can be reached and that is that on the average, English children ages 10 yr. 8 mo. to 11 yr. 7 mo. are superior in arithmetic achievement to American children of the same age." *It is imperative that interested readers study carefully the actual Buswell and Bogut reports* in order to be familiar with the details of each research study, the relation between them, and the many factors which have a significant bearing on the findings and conclusions associated with each investigation.

(Also refer to the Tracy investigation listed at a later point in this summary of research on arithmetic instruction, 1959.)

4. CASIS SCHOOL FACULTY, The. *Meeting Individual Differences in Arithmetic* (Directed and Edited by Frances Flournoy and Henry J. Otto). Bureau of Laboratory Schools, Publication No. 11. Austin: University of Texas, 1959. 184 pp.

Discusses the problem of individual differences in arithmetic in relation to the work of the classroom teacher, based in part upon research findings and implications. Presents 14 "narratives" which describe procedures and techniques for differentiated instruction on selected topics in grades 1–6, growing out of a series of try-outs and evaluations in actual classroom situations.

5. CHETVERUKHIN, NIKOLAI F. "Mathematics Education in the Soviet 7-Year School." THE ARITHMETIC TEACHER 6: 1–5; February 1959.

Presents an analysis of the mathematics program in the USSR for the seven-year period of compulsory schooling (ages 7 to 14).

6. DAVIS, O. L., JR., BARBARA CARPER, and CAROLYN CRIGLER. "The Growth of Pre-school Children's Familiarity with Measurements." THE ARITHMETIC TEACHER 6: 186–190; October 1959.

Reports data from administering the "MacLatchy Test of the Preschool Child's Familiarity with Measurement" to 23 four-year-olds enrolled in a nursery school and 29 five-year-olds enrolled in the kindergarten of a campus demonstration school. Testing was done in February. Tables are presented to show the percentages of correct responses for each of the two samples on questions about: the measurement of time; liquid measure, avoirdupois weight, and length; money and groups; and miscellaneous measures. No tests for the statistical significance of differences between percentages are reported. The authors conclude that " . . . marked differences in familiarity with measurement were noted between responses of four- and five-year-old children. While differences between the groups were found for most items, the older children had not achieved mastery of the ideas. This is consistent with previous research evidence about concept development that indicates that growth, while substantial between age groups, is continuous and, for different children, occurs at varying rates . . . to generalize from these data is dangerous. However, some conclusions may be expressed with caution. Pre-school children have some understanding of common measures. Significant growth may occur between the nursery school and kindergarten years for some measures. These findings lend encouragement to the belief that pre-school-age children may profit from direct experiences designed to foster familiarity with common measures and measurement." A bibliography of 11 references is included.

7. DE FRANCIS, JOHN. "Beginnings of Mathematical Education in Russia." THE ARITHMETIC TEACHER 6: 6–11, 16; February 1959.

Presents an analysis of a first-grade (age 7) arithmetic program in the USSR, based on a

study of *Arithmetic Textbook for the First Grade of Primary School* (*Arithmetika. Uchebnik dlya pervogo klassa nachalnoi shkoly*). Also compares this program with the program exemplified by several first- and second-grade arithmetic textbooks used here in the United States.

8. DURRELL, DONALD D., and others. "Adapting Instruction to the Learning Needs of Children in the Intermediate Grades." *Journal of Education* 142: 1–78; December 1959.

Reports a study of a modified Winnetka plan of "team learning" in 47 self-contained classrooms in eight elementary schools during the 1958–59 school year. Findings were based on data from 384 children in 16 fourth-grade classes, 448 children in 16 fifth-grade classes, and 354 children in 15 sixth-grade classes. Chapter 3 of the monograph reports the findings regarding arithmetic (classified as a "skills subject" along with reading, spelling, and language—in contrast with the "content subjects" of social studies, literature, and science). The following findings, among others, were reported: "The differentiated program in arithmetic produced statistically significant gains in problem solving in grades five and six. In computation skills there was a statistically significant gain in grade five, a slight gain in grade six. Grade four showed no change in problem solving, a slight loss in computation."

9. FELDHUSEN, JOHN F., and HERBERT J. KLAUSMEIER. "Achievement in Counting and Addition." *The Elementary School Journal* 59: 388–393; April 1959.

Sought to determine achievement levels in counting and addition that would serve as appropriate new learning tasks for children of low, average, and high intelligence having a mean chronological age of 117 months. Data were derived from 20 boys and 20 girls in each of three WISC IQ groups: low (56–81), average (90–110), and high (120–146). Achievement levels in each task are reported for each IQ group.

(See Klausmeier-Feldhusen study reported in this section; also Check and Feldhusen studies in Section II.)

10. FLOURNOY, FRANCES. "A Consideration of Pupils' Success with Two Methods for Placing the Decimal Point in the Quotient." *School Science and Mathematics* 59: 445–455; June 1959.

Reports a study comparing pupils' success with the "caret" and the "subtractive" methods for placing the decimal point in a quotient when each method is taught and learned with attention given to the basic underlying mathematical principle involved. The "caret" method was used with 71 pupils in three sixth-grade classes; the "subtractive" method, with 66 pupils in three sixth-grade classes. The "median grade-level arithmetic achievement . . . on a standardized test taken in September" was the same (5.8) for both samples. At the end of the experimental period (the starting date and length of which were not mentioned in the research report) a two-part test was administered: "Part I of the test consisted of 12 division examples involving decimal fractions. . . . Part II of the test was designed to evaluate the pupil's understanding of the method he was taught to use in placing the decimal point in the quotient." The following conclusions were among those drawn: "In general, pupils taught to make the divisor a whole number by multiplying by a power of ten and to multiply the dividend by the same number and use the caret more often correctly placed the decimal point in the quotient than pupils who were taught the subtractive method. The above-average arithmetic achievers, in the sampling of pupils taught the subtractive method, more often correctly placed the decimal point in the quotient than the above-average achievers taught the other method. However, this difference was very slight. For the below-average arithmetic achievers the subtractive method was decidedly more difficult than was the method of making the divisor a whole number . . . the nature of the subtractive method seems to provide more opportunity for error in placing the decimal point in the quotient than does the other method. . . . It is recognized that the introductory presentation given by teachers using the subtractive method may have varied much more than those presentations given by the other teachers through using the textbook as a guide. [The state-adopted textbook presents the "caret" method.] However, there was considerable indication that children taught the subtractive method understood the mathematical principle basic to it as well as pupils taught to make the divisor a whole number and to use the caret to indicate each new decimal place."

11. FLOURNOY, FRANCES. "Children's Success with Two Methods of Estimating the Quotient Figure." THE ARITHMETIC TEACHER 6: 100–104; March 1959.

Reports an exploratory study involving two fifth-grade groups who were learning to divide by 2-place divisors. One group (two classes, 61 children) was taught to use the "one-rule method" for estimating quotient digits; the other group (two classes, 63 children) was taught to use the "two-rule method." At the outset the groups were roughly comparable on computational ability in general (average grade-scores of 5.4 and 5.1 respectively). Five and one-half months after instruction was initiated, all children were given an untimed test of 10 examples involving 2-place divisors ("Five of the examples involved divisors with 5 or less in the one's place of the divisor, and five of the examples had 6 or more in the one's place of the divisor"). Various findings are presented, but none takes the form of a comparison which leads to a measure of the statistical significance of differences in performance between the two groups. In her conclusions the author states: "From observations made in this investigation it does not seem wise to *require* all children to apply a *two-rule procedure* for estimating the quotient figure." The author also states: "It may be that at some stage in their development all children should have an opportunity to learn the *two-rule procedure* and then to develop skill with the method with which they are most successful."

12. FLOURNOY, FRANCES. "Providing Mental Arithmetic Experiences." THE ARITHMETIC TEACHER 6: 133–139; April 1959.

Summarizes and discusses findings from selected studies relating to (1) the actual uses of "mental" arithmetic by children (and others), (2) the effectiveness of a planned program of instruction in "mental" arithmetic, and (3) provisions in children's textbooks for the development of ability in "mental" arithmetic. Concludes with suggestions and recommendations relative to the development of ability in "mental" arithmetic.

13. GUNDERSON, AGNES G., and MRS. ETHEL GUNDERSON. "What Numbers Mean to Young Children." THE ARITHMETIC TEACHER 6: 180–185, 190; October 1959.

Twenty years ago Miss Anges Gunderson conducted a study of "number concepts held by 7-year-olds" that has been referred to frequently in many connections since that time. This earlier study reported the ideas or concepts that a group of 17 children who had completed the first half of the second grade held with regard to the natural numbers 1–12 and the fractions one-half, one-fourth, and one-third. Now, twenty years later, Miss Gunderson and Mrs. Gunderson report findings from a similar study with a group of 26 seven-year-olds who were beginning the second half of the second grade, and compare the findings from the two studies. (The sample for the initial study involved children at the University of Wyoming Elementary School. The sample for the presently reported study involved children in an elementary school on Long Island.) The authors report that: "In comparing the responses given in this study with those of the study made 20 years ago, one notes many number concepts common to both studies. Each study reveals some ideas not shown in the other but the similiarities are far greater than the differences." They further state that: "The responses given by children in this study reflect approved practices in the teaching of arithmetic which courses in teacher education, methods courses, professional magazines, modern textbooks and accompanying teachers' manuals are striving to promote." Detailed comparisons are too numerous to include in this summary. Readers interested in such should consult the actual research report.

14. HARTUNG, MAURICE L. "Selected References on Elementary-School Instruction: Arithmetic." *The Elementary School Journal* 60: 166–168; December 1959.

The current edition of this annual *Journal* annotated listing embraces references to significant theoretical and practical discussions and to research studies relating to various phases of mathematics instruction at the elementary-school level. Twenty-two annotated references are included in this current listing.

15. ISAACS, ANN F. "A Gifted Underachiever in Arithmetic." THE ARITHMETIC TEACHER 6: 257–261; November 1959.

Presents a case study of a sixth-grade girl, almost 11 years old, with a Stanford-Binet IQ between 170 and 180 and an average reading grade of 9.4, whose arithmetic achievement in both "reasoning" and "computation" fell below the fifth-grade September norm (5.0).

16. JAROLIMEK, JOHN, and CLIFFORD D. FOSTER. "Quantitative Concepts in Fifth-Grade Social-Studies Textbooks." *The Elementary School Journal* 59: 437–442; May 1959.

Based on an analysis of three widely used fifth-grade social studies textbooks, devised a 48-item multiple-choice test of quantitative concepts which was administered to 500 fifth-graders in 15 schools. Major focus of the research report was on data for 225 of these children who had reading-grade scores of 5.0 and above and IQ's of 90 and above. For this sub-sample, found a positive relationship between scores on the test of quantitative concepts and the factors of mental ability and reading ability. In general, found that not all types of quantitative concepts present the same degree of ease or difficulty of comprehension among children. The authors stated that: "It was our feeling that social-studies textbooks use certain number concepts before they are taught in the parallel arithmetic programs in most schools. However, this study does not present evidence to substantiate this feeling. This point could profitably be studied further."

17. KEISLAR, EVAN R. "The Development of Understanding in Arithmetic by a Teaching Machine." *The Journal of Educational Psychology* 50: 247–253; December 1959.

"In this study, the problem was to explore the possibility of using a multiple-choice method for the automated teaching of 'understanding,' specifically, an understanding of areas of rectangles. By *understanding* is meant the ability to answer a variety of questions different from those encountered during training but belonging to the same general class; the broader this class is, the greater is the understanding." The material to be learned was programmed in a sequence of 110 items. "The teaching machine used in this study was an extensive adaptation of the Film Rater used by the Navy for teaching aircraft identification. Multiple-choice items on a Kodachrome strip-film were projected in sequence upon a viewing plate. The learner responded to each item by pressing one of five buttons. If the answer was correct a green light was turned on and the next item could be brought into view by pressing a special button. But if this answer was wrong a red light came on; only after turning off this red light could the learner try again. To proceed to the next item the learner had to answer correctly." Twenty-eight children were selected from the fifth- and low-sixth-grades so as to yield 14 pairs matched on the basis of sex, IQ, reading ability, and scores on a pre-test consisting of 12 problems involving multiplication and division and eight problems involving the areas of rectangles. Children in the experimental group used the teaching machine for two or three class periods on successive days, with total time ranging from an hour and a half to slightly more than two hours. Children in the control group " . . . were given no special instruction of any kind. They were used to control for the effects of incidental learning such as that which might result from the administration of the pre-test." At the end of the period of machine instruction both groups were given a post-test which consisted of 16 problems on the areas of rectangles: the same eight problems used in the pre-test plus eight others which generally were more difficult. It was found that: "Subjects performed significantly better . . . than did their matched controls who received no planned instruction on this topic." However, " . . . they learned far less than what had been expected." Possible program revisions for more effective learning were discussed in some detail.

(The reader should note with care that the children who used the teaching machine were compared *only with children who received no instruction at all*. This study did *not* compare learning via automated teaching with learning via non-automated teaching, as it were.)

18. KLAUSMEIER, HERBERT J., AND JOHN F. FELDHUSEN. "Retention in Arithmetic among Children of Low, Average, and High Intelligence at 117 Months of Age." *Journal of Educational Psychology* 50: 88–92; April 1959.

Sought to test the hypothesis that: "retention of arithmetic learning is the same among children of low, average, and high intelligence at a mean age of 117 months when the original task is graded to the learner's achievement level." Data were derived from 20 boys and 20 girls in girls in each of three WISC IQ groups: low (56–81), average (90–110), and high (120–146). Two learning tasks were studied: one involving counting, the other involving addition. On the basis of relevant end-test and retention measures, it was stated that: " . . . the researchers found no difference among means of the three IQ groups significant at the .01 level and conclude that retention is the same for the three groups as hypothesized."

(See Feldhusen-Klausmeier study reported in this section; also Check snd Feldhusen studies in Section II.)

19. KRAMER, KLAAS. "Arithmetic Achievement in Iowa and The Netherlands." *The Ele-*

mentary School Journal 59: 258–263; February 1959.

Reports a study in which the same specially designed arithmetic tests were administered to 1511 Dutch children and 1530 American children (in Iowa). "The tests were prepared by selecting and modifying items from the arithmetic section of the Iowa Tests of Basic Skills for Grades 6, 7, and 8." Test 1 consisted of 35 problem-solving items; Test 2, 59 items "... designed to measure understanding of concepts and processes of arithmetic." (Note that computational ability *per se* was not measured.) Based on comparative findings for grades 5 and 6, coupled with data for only Iowa children in grades 7 and 8, it was stated that: "On both tests, pupils in Grades 5 and 6 in Dutch schools performed considerably better than their Iowa counterparts in corresponding grades. The differences are sizable and significant at better than the 1 per cent level. In fact, average performance for the sixth grade in The Netherlands is somewhat higher than the Iowa average for eighth grade." Discusses various factors (e.g., the fact that "... almost twice as much time is devoted to formal instruction in arithmetic in the first six grades in The Netherlands as in America") that have a bearing on the meaningfulness of the study's findings and on the conclusions and implications which may be drawn therefrom.

20. LEWIS, EUNICE, and ERNEST C. PLATH. " 'Plus' Work for 'Plus' Pupils." THE ARITHMETIC TEACHER 6: 251–256; November 1959.

Reports on the nature of and results from a special instructional program used during a semester with 11 children (three fifth-graders and eight sixth-graders) having IQ's of 130 and above and arithmetic achievement grade-scores of 6.5 and above. Tentative conclusions included the following: "(1) children with high scholastic ability are capable of gaining insight into numerical relationships at a much higher level than is normally presented to them on a grade level which matches their chronological age; (2) under the guidance of a well-prepared, experienced teacher, high ability-level children are capable of developing generalizations concerning the numerical relationships even though the exact verbalization on the part of the children is not possible until later." The authors suggest that, among other things: "A critical examination should be made of the self-contained classroom to determine if it is possible for any one teacher to attain the degree of competence in all subject-matter areas necessary to provide instruction to talented students on the level they are capable of performing."

21. OLANDER, HERBERT T., and BETTY IRENE BROWN. "A Research in Mental Arithmetic Involving Subtraction." *Journal of Educational Research* 53: 97–102; November 1959.

The study involved more than 1700 children in grades 6–12 who were given three forms of a 26-item subtraction test (14 examples involving whole numbers, three involving decimal fractions, 3 involving common fractions, three involving denominate numbers, and three verbal problems); one form of the test was administered wholly orally, a second form was presented in the form of "flash cards," and the third form was presented as a "written test" for which pupils could use pencil and paper to compute. Supplementary data were collected by interviewing the five per cent of the children who were highest achievers and the five per cent who were lowest achievers, based on the oral form of the test. Numerous findings and conclusions are presented relative to: (1) proficiency in mental arithmetic in relation to the method by which examples are presented; (2) proficiency in mental arithmetic when examples are presented orally, in relation to school grade, sex, IQ, general arithmetic achievement, and memory span; (3) thought processes and methods of solution for high achievers as compared with low achievers; and (4) the relation of thought processes and methods of solution to factors such as school grade, sex, IQ, general arithmetic achievement, and memory span. Findings and conclusions appear to imply the need for more systematic, differentiated instruction relative to mental arithmetic as part of the regular arithmetic program.

22. RAPPAPORT, DAVID. "Testing for Meanings in Arithmetic." THE ARITHMETIC TEACHER 6: 140–143; April 1959.

Discusses the problem of measuring the "understanding of meanings in arithmetic," particularly through tests of the paper-and-pencil type. Includes 10 sample items from an objective pencil-and-paper test developed in connection with the author's research on the problem discussed. (Rappaport's research was reported earlier in the March 1958 issue of *The Arithmetic Teacher*, and was included in the summary of "Research on Arithmetic Instruction—1958" appearing in the April 1959 issue of *The Arithmetic Teacher*.

Studies by other researchers relating to the problem of measurement of mathematical understandings have been included in the counterpart of the present research summary for earlier years.)

23. Ruddell, Arden K. "Levels of Difficulty in Division." The Arithmetic Teacher 6: 97–99; March 1959.

Examples of the responses derived from structured interviews with 358 children (grade level or levels not specified in the research report) to illustrate definitions in behaviorial terms of four important mathematical understandings involving the process of division. These involved an understanding of division as a special case of subtraction, an understanding of division as the reverse (inverse) of multiplication, an understanding of the division algorism in relation to our decimal system of notation and the place-value principle, and an understanding of divisor-dividend-quotient relationships.

24. Schutter, Charles H., and Richard L. Spreckelmeyer. *Teaching the Third R* (A Comparative Study of American and European Textbooks in Arithmetic). New York: Council for Basic Education, 1959. 46 pp.

This monograph compares the mathematical content of American and European arithmetic programs in the "elementary" grades, based on an analysis of children's textbooks. Based on their findings, the authors present a set of five recommendations which, they hope, will " . . . first, receive the benefit of additional research; and, second, lead eventually to the preparation of text and related materials for implementing an improved program in arithmetic."

25. Sister Josephina. "Differences in Arithmetic Performance." The Arithmetic Teacher 6: 152–153, 166; April 1959.

Reports a study in which a non-standardized arithmetic test, consisting of 30 computational examples and 20 problems, was administered to 122 fifth-grade-children at the end of the school year in June and then again at the beginning of the new school year in September, following the three-month summer vacation. The mean score on the 30 computation examples dropped from 20.91 in June to 15.09 in September, and the mean score on the 20 problems dropped from 13.39 in June to 9.36 in September. Both loses were found to be statistically significant beyond the one per cent level of confidence. Data also are reported relating to the "per cent of failure" on various categories of test items at both the June and September testings.

26. *Studies in Mathematics Education:* A Brief Survey of Improvement Programs for School Mathematics. Chicago: Scott, Foresman and Co., 1959. 57 pp.

Includes summaries of the work, etc. of experimental curricular projects in elementary-school mathematics. (The bulk of the monograph is devoted to the much greater number of experimental curricular projects in mathematics above the elementary-school level.)

27. Tracy, Neal H. "A Comparison of Test Results: North Carolina, California and England." The Arithmetic Teacher 6: 199–202; October 1959.

Reports a study based on administering the "Buswell test" to two groups of North Carolina children: 194 whose CA ranged from 10 yr. 8 mo. to 11 yr. 7 mo., and 555 eighth-graders. Findings were compared with those from the Buswell investigation. (See G. T. Buswell, "Comparison of Achievement in Arithmetic in England and Central California," *The Arithmetic Teacher* 5: 1–9; February 1958.) The author states that: " . . . the North Carolina 10–8 to 11–7 age group scored significantly higher than the California urban group although still significantly lower than the English urban group." Furthermore: "The comparison between the groups at the point of terminal training in arithmetic, that is the English urban group, 10 years, 8 months, to 11 years, 7 months, with the North Carolina urban group of eighth graders . . . indicates that there is no significant difference in levels of achievement in the two groups as measured by the total test." Also: "There is the undeniable fact that, as measured by this particular test, American schools are taking two extra years in which to assure achievement levels equivalent to those attained in the English schools." *It is imperative that interested readers study carefully the actual Buswell and Tracy reports* in order to be familiar with the details of each research study, the relation between them, and the many factors which have a significant bearing on the findings and conclusions associated with each investigation.

(Also refer to the Bogut investigation listed at an earlier point in this summary of research on arithmetic instruction, 1959.)

28. Weaver, J. Fred. "Research on Arithmetic Instruction—1958." The Arithmetic Teacher 6: 121–132; April 1959.

This is the counterpart of the current summary, covering the 1958 calendar year.

29. WEAVER, J. FRED, and CLEO FISHER BRAWLEY. "Enriching the Elementary School Mathematics Program for More Capable Children." *Journal of Education* 142: 1–40; October 1959.

Discusses the problem of mathematically talented children in the elementary school, based in part on both stated and implied research findings. (Also includes illustrations of a particular type of enrichment activity developed through classroom try-outs, evaluations, and subsequent revisions.) Includes a bibliography of more than 70 citations.

30. WIRSZUP, IZAAK. "Current School Mathematics Curricula in the Soviet Union and Other Communist Countries." *The Mathematics Teacher* 52: 334–346; May 1959.

Analyzes the Russian curricular pattern in mathematics for grades 1–10 in relation to factors such as content, time allotment for class instruction, and time allotment for homework.

Section II: Doctoral Theses and Dissertations

The annotated references in this section are restricted to doctoral-level investigations that were reported in *Dissertation Abstracts* during the 1959 calendar year (regardless of the year in which the degree was awarded). Admittedly, this restriction excludes the few research studies at the doctoral level that, for one reason or another, never are reported in *Dissertation Abstracts*. Unfortunately, however, it has proven to be impractical for the writer to attempt to compile a reliable listing of such studies on an annual basis.

Some of the investigations cited here either have been reported, or will be reported, in one form or another in the periodical literature or the like.

1. ADKINS, BRYCE E. "A Topical Listing and Explanation of Selected Instructional Aids in Arithmetic." *Dissertation Abstracts* 19: 1609; January 1959. (L. C. Card No. Mic 58-5799).

Developed an annotated listing of instructional aids in arithmetic, classified under the following categories: Counting Devices, Place Value Devices, Recreational Aids, Casting Out Nines, Instructional Games, Basic Processes, The Model Store, Manipulative Arithmetic, Games for Use Outside the Classroom, Arithmetic Bulletin Boards, and Commercial Arithmetic Games.

2. ALEXANDER, VINCENT EUGENE. "The Relationship of Selected Factors to the Ability to Solve Problems in Arithmetic." *Dissertation Abstracts* 20: 1221; October 1959. (L. C. Card No. Mic 59-3057)

Using test data derived from 623 seventh-grade pupils, applied correlation techniques to investigate the relation between problem solving ability in arithmetic and each of a variety of possibly relevant factors. Identified 11 factors that appeared to be related closely to arithmetic reasoning abilities and six factors that did not appear to be related closely to this ability. Found non-significant sex differences in the ability to solve arithmetic problems. On the basis of findings, formulated a set of seven conclusions, leading to three major educational implications.

3. BURNS, PAUL CLAY. "Use of Intensive Review as a Procedure in Teaching Arithmetic." *Dissertation Abstracts* 19: 3168–3169; June 1959. (L. C. Card No. Mic 59-1699)

Data for this study were derived from 14 sixth-grade classes—seven experimental classes and seven control classes. The control classes followed "usual review procedures" in connection with the fundamental operations with common and decimal fractions; the experimental classes used specially developed study lessons and guides. Provisions were made for pre-testing, for measuring immediate recall at the close of the experimental periods (one in relation to work with common fractions, the other in relation to work with decimal fractions), and for measuring delayed recall after two weeks of interpolated activity. "The mean gain between the pre-tests and the post-tests . . . indicated a significant difference in favor of the experimental group. . . ." Recommended that consideration be given to use of the experimental procedure on a wider scale.

4. CARLOS, CARMEN BALDERRAMA. "A Study of Some Basic Guiding Principles in Teaching Selected Aspects of Elementary Arithmetic with Implications for Educational Practice and Teacher Education in The Philippines." *Dissertation Abstracts* 19: 2860–2861; May 1959. (L. C. Card No. Mic 58-5203)

Developed a check-list of 98 "generally accepted principles" underlying selected aspects of elementary-school arithmetic instruction, which

was then reduced to a set of 25 principles selected a jury as "essential" in relation to the development of problem solving abilities and skills in the fundamental operations. On the basis of these 25 "essential" principles, formulated proposals for improving educational practice and teacher education in the Philippines. Seventeen conclusions were derived from the findings.

5. CHECK, JOHN FELIX. "A Study of Retention of Arithmetic Learning with Children of Low, Average, and High Intelligence at 127 Months of Age." *Dissertation Abstracts* 20: 955–956; September 1959. (L. C. Card No. Mic 59-3176)

Sought to test the following hypothesis: ". . . that retention is the same among children of low, average, and high intelligence when the original task for each child is graded to his present achievement level." Data were derived from 20 boys and 20 girls in each of three WISC IQ groups: low (55–80), average (90–110), and high (120 and up). Two learning tasks were studied: one involving subtraction examples, the other involving problem solving. On the basis of relevant end-test and retention-measures, it was concluded that: "The hypothesis was supported that retention is the same for children of low, average, and high intelligence at a mean chronological age of 127 months when the original task for each child is graded to his achievement level."

(See Feldhusen study reported in this same section; also Feldhusen-Klausmeier and Klausmeier-Feldhusen studies reported in Section I.)

6. FELDHUSEN, JOHN FREDERICK. "A Study of Efficiency of Learning and Retention in Arithmetic among Children of Low, Average, and High Intelligence at a Mean Age of 112 Months." *Dissertation Abstracts* 19: 1651–1652; January 1959. (L. C. Card No. Mic 58-7480)

Sought to test the hypotheses that: (1) "Rate of learning of an arithmetic task is the same among children of low, average, and high intelligence," and (2) "Retention of arithmetic learning is the same among children of low, average, and high intelligence when the original task is graded to the learner's achievement level." Data were derived from 20 boys and 20 girls in each of three WISC IQ groups: low (55–80), average (90–110), and high (120 and above). Three learning tasks were studied: one involving counting, a second involving addition, and a third involving problem solving. On the basis of relevant end-test and retention measures, it was concluded that: "In learning a common problem-solving task in arithmetic low-IQ children took significantly more time that average- and high-IQ children but the difference between average and high was not significant. No significant difference among children of low, average, and high intelligence was found in retention of counting and addition tasks, graded to the learner's achievement level."

(See Check study reported in this same section; also Feldhusen-Klausmeier and Klausmeier-Feldhusen studies reported in Section I.)

7. FOLSOM, MARY O'HEARN. "A Study of Manual Material in the Field of Arithmetic." *Dissertation Abstracts* 19: 1671–1672; January 1959. (L. C. Card No. Mic 58-5817)

Major purposes of this three-fold investigation were: (1) to study the use actually made of arithmetic manuals by classroom teachers; (2) to secure the judgments of these teachers regarding the materials in the manuals used; and (3) to obtain the reactions of both experienced and prospective teachers to three specimens of manual material, each written for the same arithmetic textbook page but differing in manner of presentation, kind of suggestions offered, and the like. Data for (1) were obtained by the investigator's personal visits to 22 sixth-grade classrooms in nine different Iowa school systems. Data for (2) were obtained through a questionnaire completed by the teachers visited in (1). Data for (3) were obtained from questionnaires completed by 48 experienced teachers and 223 prospective teachers in arithmetic methods classes at five institutions of higher learning. Findings are reported from each phase of the complete investigation, along with consequent conclusions. The investigator's final statement is provocative: "It is recommended that authors of arithmetic textbooks and manuals plan the two simultaneously instead of forcing manual suggestions to fit a text page that has been written previously, and that more pre-book activities be incorporated in arithmetic manuals since the overwhelming approval given to this feature by the subjects participating in this study was the most significant fact revealed."

8. HUDGINS, BRYCE BYRNE. "The Effects of Initial Group Experience upon Subsequent Individual Ability to Solve Arithmetic Problems." *Dissertation Abstracts* 19: 2851–2852; May 1959. (L. C. Card No. Mic 59-1346)

This study sought to test three hypotheses: "(1) Individual ability to solve arithmetic prob-

lems will be improved more as a function of initial group experience than it will as a function of individual experience. (2) Individual ability to solve arithmetic problems is improved as a function of specifying the steps involved in arriving at solution. (3) The improvement of individual problem solving as a result of group experience is a function of the relevancy of intragroup communications to the processes involved in problem solution." The experimental design involved 128 fifth-graders: four groups of 32, matched on intelligence and arithmetic ability. Each group engaged in problem solving work under one of four different experimental conditions. It was found that: " . . . subjects who worked in groups solved more problems correctly than did subjects who worked individually. This difference was significant beyond the .01 level." This finding applied immediately after a three-day experimental period. However, when measures of permanence were taken one, two, five, and twelve days later the " . . . results indicated that the arithmetic problem solving ability of subjects who had worked as members of groups . . . was no greater than that of subjects who had worked individually. . . . Likewise, . . . subjects who had been trained in the use of specification were not superior to those . . . who had worked under non-specification conditions. . . ."

9. LAWSON, JOHN HERBERT. "The Construction and Revision of an Arithmetic Vocabulary Test for Grades Four, Five, and Six." *Dissertation Abstracts* 19: 3246–3247; June 1959. (L. C. Card No. Mic 59-440)

Based on an analysis of quantitative vocabulary in arithmetic textbook series and on item try-outs with over 1000 children in grades 4, 5, and 6, developed two balanced forms of an arithmetic vocabulary test for the intermediate grades. Each final 80-item test form had an estimated reliability of .91, estimated means of 53.75 and 54.33, and estimated sigmas of 11.00 and 11.15; indices of item validity, determined in relation to Flanagan's table, ranged from .720 to .430.

10. LENO, RICHARD STANLEY. "Children's Methods of Problem Solving in Arithmetic." *Dissertation Abstracts* 19: 2549; April 1959. (L. C. Card No. Mic 59–242)

Data for the study were gathered from 886 pupils at several unspecified grade levels. Three matched groups were established on the basis of scores on a standardized test of problem solving. Children in one of the groups (the experimental group) were interviewed to determine the ways in which they solved test problems, and this information was used as the basis for systematic instruction in problem solving with these children. One of the other two groups, which served as controls, engaged in the study of problem solving using a standard method devised by the investigator. The third group apparently had no systematic instruction in problem solving. Upon administration of a second form of the standardized test at the end of the experimental period, it was found that: "In no grade, chronological age level, or mental ability level was there a consistent indication that the experimental method of teaching problem solving gave significantly greater gains than were obtained in either of the control groups. All groups made gains that were approximately five times greater than normal expectancy for the elapsed time. The fact that approximately equal gains were made in all groups suggests that whatever factors contributed to these gains were not applied solely to the experimental group."

11. MILLER, MELVIN LINDER. "Effects of Different Types of Kindergarten Programs upon Reading and Arithmetic Readiness." *Dissertation Abstracts* 19: 2029–2030; February 1959. (L. C. Card No. Mic 58–1723)

Used six sections of kindergarten children, separated into three treatment groups, to test the the general null hypothesis that " . . . there was no significant differences in the mean accomplishment on readiness tests of kindergarten groups with three different treatments in a reading readiness program and a number readiness program," whether measured immediately upon conclusion of the experimental period or after the intervening summer vacation. The three programs were characterized as follows: "On treatment group followed a treatment method which placed no special emphasis on reading and number readiness activities. The second treatment group incorporated a special emphasis on reading and number readiness into the normal on-going kindergarten activities. The third treatment group utilized selected commercial materials in reading and number readiness in the manner prescribed by the authors of the materials. Special and selected materials, methods, and activities were utilized by this group." Based on data from relevant testing, it was found that: " . . . there was no significant difference in the mean ac-

complishment in either reading readiness or number readiness at the close of the treatment period of three kindergarten groups that have had different treatments in their reading readiness and number readiness program." Similar non-significant differences were found after the summer vacation.

12. MONTGOMERY, CLYDE RAYMOND. "An Investigation of the Learning of the Three Cases of Percentage in Arithmetic." *Dissertation Abstracts* 19: 1676–1677; January 1959. (L. C. Card No. Mic 58-5846)

Data derived from the administration of a 240-item test battery to seventh-grade pupils from 20 schools. The " . . . test battery was so constructed that each of the three factors of the percentage situation—areas, rate, and type of items—could be investigated in turn, while the other factors were held constant." Overall: "The seventh-grade pupils solved 71.1% of the items correctly, 20.7% incorrectly, and omitted 8.3% of the items." Furthermore: "The correct responses were more numerous in Case I than in Case II, and in Case II than in Case III." Numerous other findings are reported, with accompanying conclusions.

13. MOTT, EDWARD RAYMOND. "An Experimental Study Testing the Value of Using Multisensory Experiences in the Teaching of Measurement Units on the Fifth and Sixth Grade Level." *Dissertation Abstracts* 20: 1678–1679; November 1959. (L. C. Card No. Mic 59-5120)

"This study was designed for the purpose of measuring certain experimental methods of teaching, namely, a prearranged system of multisensory materials. The particular areas of attitude, quantitative understanding, and computation ability were studied. Pre and post tests were administered to both experimental and control pupils. . . . The control section composed of 157 pupils in six classes proceeded as usual throughout the school year. No attempt was made to influence this group as to time, method, or aids used. Multisensory aids were used at every opportunity during arithmetic measurement classes in the experimental section which was made up of 70 children in two classes." Reported findings led to the following statement: "This study shows that significant gains resulted under both experimental and control conditions. In spite of the fact that the experimental group did not gain significantly over the control group on the three areas tested, this should not be construed to mean that the experimental methods should not be used."

14. PHILLIPS, CLARENCE ALOIS. "The Relationship Between Achievement in Elementary Arithmetic and Vocabulary Knowledge of Elementary Mathematics as Possessed by Prospective Elementary Teachers." *Dissertation Abstracts* 20: 1687–1688; November 1959. (L. C. Card No. Mic 59-4550)

"The problem of this study was to determine the degree of relationship between arithmetic achievement and vocabulary knowledge of elementary mathematics as possessed by prospective elementary teachers. . . . A total of 52 students enrolled in Mathematics 202 (Arithmetic for Teachers) at the University of Illinois (Urbana, Illinois) was used in the study." Three tests were administered to each student: the California Short-Form Test of Mental Maturity, a specially constructed 40-item achievement test in elementary arithmetic, and a 15-item recall-type test of arithmetic vocabulary. Numerous correlation coefficients based on test scores are reported.

15. SISTER MARIE CONSTANCE DOOLEY. "The Relation between Arithmetic Research and the Content of Elementary Arithmetic Textbooks, 1900–1957." *Dissertation Abstracts* 20: 562–563; August 1959. (L. C. Card No. Mic 59-2611)

Based on a study of relevant research since 1900 and on a study of 153 arithmetic textbook series published in the United States since 1900, sought to determine the effect of research upon content and method as reflected in children's textbooks. Found nine instances in which the research effect was "direct and immediate," and three instances in which research recommendations were "rejected to some degree." In general it was stated that: "Incorporation [of research findings] was found to have been rapid when the recommendations were clear, concise, and exact. Trends took longer to develop when the recommendations were general, intangible, or based upon subjective data. With one exception, the use of concrete materials above the primary grades, recommendations published in yearbooks tended to be applied quickly."

16. SPENCE, EUGENE SAMUEL. "Intra-Class Grouping of Pupils for Instruction in Arithmetic in the Intermediate Grades of the Elementary School." *Dissertation Abstracts* 19: 1682; January 1959. (L. C. Card No. Mic 58-5635)

"The purpose of this research was to study intra-class grouping of pupils for instruction in arithmetic in the intermediate grades of the elementary school. The objectives were to discover the various techniques used by teachers for grouping pupils; various methods used in teaching the subgroups; attitudes of parents, pupils and teachers toward subgrouping; and to determine the effects of subgrouping on arithmetic achievement." Of the 767 pupils involved in the experiment, 567 were in experimental classrooms in which children were grouped in three subgroups; the remaining 200 pupils were in control classrooms where whole-class teaching was used. "All pupils were given a group type intelligence test at the beginning of the study, and they also were given different forms of a standardized arithmetic achievement test during the 4th, 15th, and 34th weeks of the school year." Various findings were reported, including this one: "The arithmetic test results showed that pupils in the experimental classes achieved higher scores than did pupils in the control classes," the difference being significant at the 1% level at all grades.

17. WILSON, GILBERT M. "Quantitative Content in Elementary School Social Studies Textbooks." *Dissertation Abstracts* 19: 2816; May 1959. (L. C. Card No. Mic 59-1122)

"The major purpose of this study was to determine the nature and frequency of definite quantitative terms which appear in elementary school social studies textbooks and to determine whether or not these terms appear in arithmetic textbooks at corresponding grade levels. . . . One history, one geography, and two arithmetic texts for each of the Grades IV through VII were analyzed for the purposes of this investigation . . . Cumulative totals indicate that 73,262 uses of definite quantitative terms are found in the social studies books. . . . When the terms found in the social studies books were checked for their appearance in the arithmetic books, it was found that, with certain exceptions, they appear generally at corresponding grade levels. The most notable exceptions occur in such categories as arithmetic symbols, statistical terms, and words bearing arithmetic connotation."

18. WOLF, WILLIAM CHARLES, JR. "An Evaluation of Non-Pencil-and-Paper Materials Prepared for Use in the Elementary School Arithmetic Program." *Dissertation Abstracts* 20: 2170; December 1959. (L. C. Card No. Mic 59-5742)

This study sought to determine the effectiveness of two different pedagogical procedures for presenting five non-pencil-and-paper lessons on addition to fourth- and fifth-grade children as a supplement to their regular classroom instruction. One procedure utilized filmed lessons which were supplemented by the classroom teacher; the other procedure utilized printed materials presented entirely by the classroom teacher. Findings from an analysis of relevant data led the investigator to conclude that of the two pedagogical procedures, the one involving use of a film presentation was to be preferred on each of several bases for comparison.

Areas of Research Interest

What aspects or phases of arithmetic were represented in the research reported during the 1959 calendar year? The table below attempts to answer this question in a concise way which also indicates the relative amount of research reported in each area of interest. (In one instance, however, several studies listed separately are actually parts of the same major research project.)

Some of the references might have been classified in several ways. However, each published research report has been placed in just one category in the table which is printed at the top of the next page.

The numerals in the right-hand columns are not frequencies, but refer to specific research reports in each section of the basic listing.

Concluding Statement

Published research during the 1959 calendar year has touched upon some "familiar" areas of research interest: problem solving, mathematical understandings, mental arithmetic, audio-visual aids, number abilities of young children, and the like.

Research interest in certain areas is definitely on the increase. Note the studies relating to individual differences, particularly in relation to the talented; also note the studies relating to arithmetic programs in foreign countries, especially the USSR, and to levels of arithmetic achievement in foreign countries as compared with the U. S.

CLASSIFICATION OF RESEARCH TOPICS

Area	*Section I*	*Section II*
Provisions for individual differences (general)	4, 8	16
Studies re children of different IQ levels	9, 18	5, 6
Arithmetic for the talented	15, 20, 29	—
Remedial arithmetic	2	—
Arithmetic programs in foreign countries	5, 7, 24, 30	4
Arithmetic achievement in U. S. and other countries	3, 19, 27	—
Problem solving	—	2, 8, 10
Instructional aids in arithmetic	—	1, 13
Use of teaching machines in arith. instruction	17	—
Non-pencil-and-paper arithmetic	12, 21	18
Number abilities of young children	6, 13	11
Measuring aspects of arith. understanding	1, 22	9, 14
Children's texts and teachers' manuals	—	7, 15
Aspects of division	10, 11, 23	—
Quantitative concepts in social studies	16	17
Knowledge of percentage	—	12
Intensive review	—	3
Change in achievement over summer months	25	—
Research summaries, reviews, etc.	14, 26, 28	—

Note well the presence of a research study on the use of teaching machines. We can expect many more research reports on automated learning in arithmetic during the next few years.

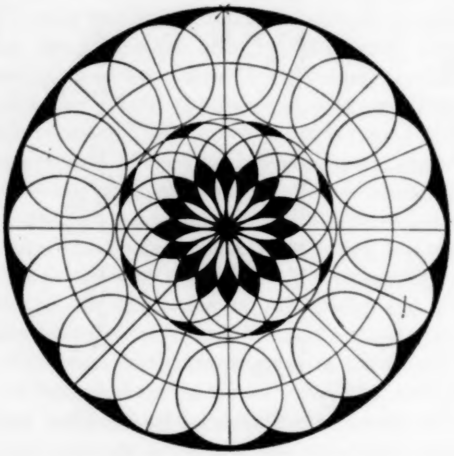
We also can look forward to research reports that will grow out of "experimental" programs on arithmetic currently in operation or just getting under way, such as: The University of Illinois Arithmetic Project, The Madison Project (Syracuse University), the Stanford Project, and the School Mathematics Study Group's new project at the elementary-school level. Descriptive accounts of some of these already are in our literature. Research reports soon will follow.

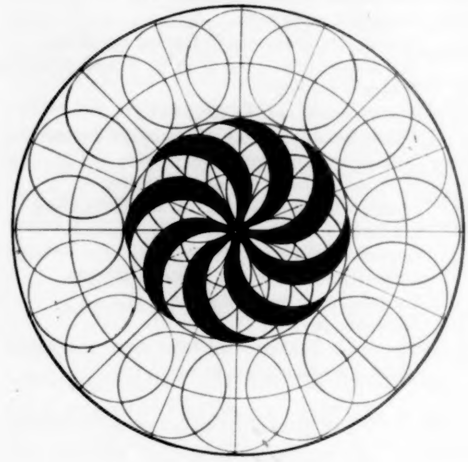
The next few years can be exciting and fruitful years for research on arithmetic if we but make them such.

Corrections

1. In the January issue on page 24, column 2, Camper 7 should have used 5280 feet in his mile and corrected the eighth of a mile.
2. In the February issue on page 74, column 1, paragraph 3, the word "reserved" should have been "reversed."
3. In the February issue on page 75, Table VI has some errors which the seventh grade in the Edison Jr. H. S. of Milwaukee discovered. A little multiplication will reveal them.
4. In the February issue the name Pauline Dubinsky should have been Pauline Dubitsky.

A Recipe for Angle, Circle, Construction Surprise





EQUIPMENT:

Ruler, compasses, sharp pencil, colored pencils, alert pupil.

INGREDIENTS:

Concentric circles, diameter, radius, bisector, semicircle, quadrant, intersecting circles, neatness, accuracy, concentration, careful workmanship.

PROCEDURE:

Describe 3 concentric circles having a 1 in., a 3 in., and a 4 in. radius. Draw the diameter of the large circle. (This is also the diameter of the 2 smaller circles.)

Bisect the diameter. Use this bisector as another diameter of the large circle.

Next bisect each quadrant. Then bisect each resulting 45° angle.

Now there are 16 bisectors.

Carefully place the compass point on the circumference of the 1 in. circle at the point at which a bisector cuts this circumference.

Describe a 1 in. circle. Do the same at each of the succeeding bisectors. Repeat this same procedure at the point at which the bisectors cut the 3 in. circle's circumference.

Again there will be 16 intersecting circles. Now with colored pencils put the frosting on the Surprise. Using only 3 or 4 colors, carefully color the design. All started with the same basic receipe but the surprise is in the kaleidoscopic results.

RESULTS:

A better understanding of simple geometric terms.

Practice in following directions.

Satisfaction from a job well done.

Fun with mathematics. Shading corresponding parts with colored crayons makes interesting designs.

Contributed by

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Nine and Ten to the Eight-year Old

In the October, 1959 issue of THE ARITHMETIC TEACHER, Agnes and Ethel Gunderson reported on "What Numbers Mean to Young Children." Miss Annie Roberts of Edmonton, Canada supplements the list of concepts involving Nine and Ten. These are printed to show additional levels of understanding. The list could be greatly increased from many sources. The formation of concepts depends in large measure upon how the home and the school stimulate learning in relation to the child's surroundings.

Nine

9 blocks away.

9 pennies is 1 short of a dime.

My dad says that our cat has 9 lives, but I don't really believe it has.

9 is $\frac{3}{4}$ of the way around the clock.

9 is $\frac{3}{4}$ of a dozen. $\frac{3}{4}$ of a foot is 9 inches.

9 months is $\frac{3}{4}$ of a year. $\frac{3}{4}$ hours is $\frac{3}{4}$ of half a day.

9 twos is the same as 2 nines.

9 is 6 less than 15.

9 is 11 less than 20.

$\frac{1}{2}$ of 9 inches is $4\frac{1}{2}$ inches.

$\frac{1}{4}$ of a yard is 9 inches.

If a man was 9 feet tall he would be a giant. Was Goliath 9 feet tall?

3 threes are 9.

Ten

10¢ is 1 dime. $\frac{1}{4}$ of 10 is $2\frac{1}{2}$. $\frac{1}{2}$ of 10 is 5.

Ten dollar bill.

10 dimes in a dollar.

10 sticks of gum in 2 packages.

Everything goes in tens—10 ones are 10, 10 tens are 100, 10 hundreds are 1000. (An argument ensued as to whether 10 thousands would be a million or not.)

Our bell rings at 10 to nine.

In winter we only get about 10 minutes recess because it takes so long to dress.

Our wing has 10 classrooms. So has the west wing.

10 fingers and 10 toes.

10 provinces in Canada.

10 commandments in the Bible.

My mom had her tenth anniversary.

5 pairs of shoes.

Our carboard rulers are 10 inches long.

We go to school 10 months.

If you think of 10 it is easy to work with 9.

You have 1/10 of the week when you don't have to work. (I have $\frac{1}{2}$ day for administrative duties.)

5 quarts = 10 pints.

10 quarts = 20 pints.

I like to count by 10's.

Ten is 1 more than 3 threes, 2 more than 2 fours.

Book Review

Yes, Math. Can Be Fun! (Teacher Edition) Louis Grant Brandes. Portland, Maine: J. Weston Walch, Publisher, 1960. Paper, iv+263 pp.

If you have forgotten an interesting arithmetic or geometric puzzle and have since had the desire to recall it, you will most likely find it here. Mr. Brandes has assembled a rich collection of elementary puzzles and pastimes, most of which have a mathematical flavor.

The reviewer, as well as the author, does not recommend this book as a text for a mathematics course at any grade level. It is recommended, however, as a source for supplementary material at almost all levels. The high school and grade school teacher can find here a wide variety of topics to help stimulate interest in mathematics. To use it otherwise might defeat its purpose.

There are sections which lend themselves to further mathematical exploration. For example, in Part I, "Number Oddities," there are some numerical relationships and arithmetic short cuts which could be used to introduce or discover some interesting algebraic relationships. Later we find some individual and group projects, including the construction of some geometric linkages, i.e., the pantograph, an angle trisector, etc., which can be used in conjunction with geometrical studies.

Perhaps some of its stronger features are to be found in the variety of material and its easy reading style. This variety includes, in addition to those previously mentioned, logic puzzles, arithmetical games, optical illusions, historical reflections, etc. The clear presentation makes it suitable for individual student use and especially useful for a mathematics club.

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The Volume of a Sphere

ASSUMPTIONS:

1. The pupils understand the meaning of volume.
2. The pupils know how to find the volume of a cylinder.
3. The pupils know how to use exponents.
4. The pupils know how to use formulae.

MATERIALS NEEDED:

1. A graduate.
2. A cylinder whose inside diameter and height are equivalent to that of a sphere being used.
3. A tray.
4. Water.

DEVELOPMENT (GRADE 8):

"Class, what would happen if I were to submerge this ball into this filled cylinder? (Do it.) Yes, some water spills out. How much water spills out? That is correct. An amount equal to the volume of the sphere. Incidentally, about 200 years before Christ was born a Greek by the name of Archimedes discovered this principle while taking a bath.

"To-day we are going to develop a formula for finding the volume of a sphere. You've already told me that this sphere would displace its own volume when submerged in water. You will note that this sphere just fits into this cylinder. The inside diameter of the cylinder and its inside height are both equivalent to the diameter of the sphere. John would you fill the cylinder with water. Pour the water into the graduate and record on the board the amount of water. Thank you John. We now know the volume of the cylinder. Jane place the sphere into the cylinder. Hold it down with a pencil point and again fill the cylinder with water. Now measure the water by pouring it into the graduate. Record your reading on the board.

"Class, how does the volume of the sphere compare with that of the cylinder? Yes, the volume of the sphere is two-thirds that of the cylinder. What is the formula for finding the volume of a cylinder? Yes, $V=\pi r^2h$. So the volume of the sphere would be $\frac{2}{3}\pi r^2h$. Note that the height of the cylinder is twice the radius of the sphere. If we replace the (h) in the formula with 2 r's ($2r$) we have $\frac{2}{3}\pi r^2\cdot 2r$. Mary can you simplify this formula? That's fine Mary. We have $V=\frac{4}{3}\pi r^3$ which is the formula for finding the volume of a sphere. You will note that only one dimension is needed. That is the radius of the sphere."

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Book Review

Qualifications and Teaching Loads of Mathematics and Science Teachers, U. S. Office of Education Circular No. 575, Kenneth E. Brown and Ellsworth S. Obourn, U. S. Government Printing Office, 1959. $0.70.

Although this study is based upon the grade levels seven through twelve and the data represents the three states of Maryland, New Jersey, and Virginia, there is a good deal of information that will be of interest to teachers at all levels. Inasmuch as there is a good deal of ferment in the area of mathematics, it is interesting to note that one-fifth of the teachers received their training before 1950 and that 39% of the teachers had not taken as much mathematics as a course in calculus. On the average the mathematics teachers have 23 semester hours in mathematics and 31 semester hours in education. What conclusion might one draw: "Training in mathematics is less important than training in education," or "It is easier to learn mathematics after graduation and on the job than it is to learn the principles of educational theory and practice," or "Might it be a little easier to obtain credits in education than in mathematics?"

It would be interesting to have comparable data from other sections of the country.

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